ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Spring

FINAL EXAMINATION

21.05.2019, 15:00

-SOLUTIONS-

Question	Grade	Out of
1		12
2		12
3		16
4		12
5		15
6		18
7		20
Total		105

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 100 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 7 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions. Indicate the solutions clearly.

(6 pts.) **a)** $\left|\dfrac{5x-3}{2}\right| > 4$

$\dfrac{5x-3}{2} > 4$ or $\dfrac{5x-3}{2} < -4$

$5x-3 > 8$ $\quad$ $5x-3 < -8$

$5x > 11$ $\quad$ $5x < -5$

$x > \frac{11}{5} \Rightarrow \left(\frac{11}{5}, \infty\right)$ $\quad$ $x < -1 \Rightarrow (-\infty, -1)$

$\boxed{\text{Soln. set: } (-\infty,-1) \cup \left(\tfrac{11}{5}, \infty\right)}$

(6 pts.) **b)** $\dfrac{2}{x^2-9} = \dfrac{1}{x-3} + \dfrac{1}{x+3} = \dfrac{(x+3)+(x-3)}{x^2-9} = \dfrac{2x}{x^2-9}$

$\Rightarrow 2 = 2x \Rightarrow x = 1 \Rightarrow \boxed{\text{Soln. set: } \{1\}}$

2. (6 pts.) **a)** Solve the equality: $\underbrace{\log_3(x)}_{x>0} + \underbrace{\log_3(x-2)}_{x>2} = \underbrace{\log_3(x+10)}_{x>-10}$ $\quad \underbrace{x>0, x>2, x>-10}_{x>2}$

$\log_3|x(x-2)| = \log_3|x+10| \Rightarrow x^2 - 2x = x + 10$

$\Rightarrow x^2 - 3x - 10 = 0 \Rightarrow (x-5)(x+2) = 0 \Rightarrow \boxed{x=5 \text{ or } x=-2}$

But $x>2$ should be satisfied for the terms in the original equation to be defined $\Rightarrow$ So; $\boxed{x=5 \text{ is the only solution.}}$

(6 pts.) **b)** Evaluate: $\log_2(\sqrt{8}) - \log_5\left(\dfrac{1}{\sqrt[3]{25}}\right) + \dfrac{1}{6}\ln e^5 = ?$

$\log_2 2^{3/2} - \log_5 5^{-2/3} + \frac{5}{6}\underbrace{\ln e}_{1} = \frac{3}{2}\underbrace{\log_2 2}_{1} - \left(-\frac{2}{3}\right)\underbrace{\log_5 5}_{1} + \frac{5}{6}$

$= \frac{3}{2} + \frac{2}{3} + \frac{5}{6} = \dfrac{3(3)+2(2)+5}{6}$

$= \dfrac{18}{6} = \boxed{3}$

3. For the function $f(x) = 2x^2 - 6x - 20$:

(7 pts.) **a)** Find the vertex, x-intercept and y-intercept points

$$f(x) = 2(x^2 - 3x - 10) = 2\left(\left(x - \tfrac{3}{2}\right)^2 - \tfrac{49}{4}\right) \Rightarrow f\left(\tfrac{3}{2}\right) = -\tfrac{49}{2}$$

Vertex: $\left(\frac{3}{2}, -\frac{49}{2}\right)$

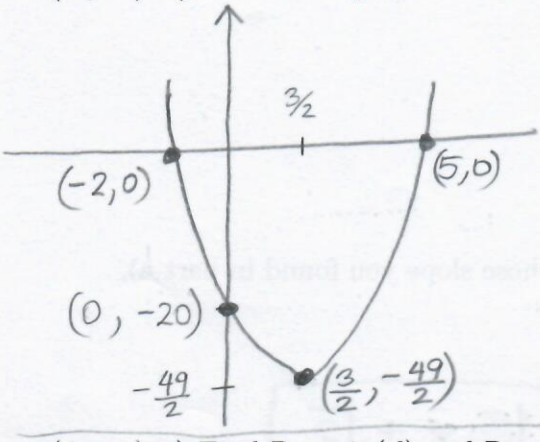
$x = 0 \Rightarrow f(0) = -20 \Rightarrow \boxed{(0,-20) \to y\text{-intercept}}$

x-intercept(s): $y = 0$

$\Rightarrow 2(x-5)(x+2) = 0$

$\Rightarrow \boxed{(-2,0), (5,0) \to x\text{-intercepts}}$

(5 pts.) **b)** Sketch the graph of the function



(4 pts.) **c)** Find Domain(f) and Range(f)

Domain: $\mathbb{R}$

Range: $\left[-\frac{49}{2}, \infty\right)$

(12 pts.) **4. a)** Find the points of discontinuity (if any) of the function

$$f(x) = \begin{cases} e^x + 3 - x^2, & \text{if } x < 0 \\ 3x - 2, & \text{if } 0 \le x < 1 \\ 2 - x^2 + \ln x, & \text{if } 1 \le x \end{cases}$$

Explain your answer in detail.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (e^x + 3 - x^2) = e^0 + 3 - 0 = 4$$
$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3x - 2) = 3(0) - 2 = -2$$

$4 \neq -2 \Rightarrow \lim_{x \to 0} f(x) = \text{d.n.e.}$

$\Rightarrow \boxed{f \text{ is discontinuous at } x = 0}$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3x - 2) = 3 - 2 = 1$$
$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2 - x^2 + \ln x) = 2 - 1 + \cancel{\ln 1}^{\,0} = 1$$

$\Rightarrow \boxed{\lim_{x \to 1} f(x) = 1}$

$\Rightarrow$ only $x = 0$ is the point of discontinuity

5. Given the curve $y = \sqrt{x+2}$,

(5 pts.) a) Find the slope of the tangent line to the above curve at the point where $x = 0$.

$$y' = \frac{1}{2\sqrt{x+2}} \Rightarrow \text{slope} = y'(0) = \frac{1}{2\sqrt{0+2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

(5 pts.) b) Find the equation of the tangent line whose slope you found in part a).

$$y(0) = \sqrt{0+2} = \sqrt{2}$$

$$y - \sqrt{2} = \frac{1}{2\sqrt{2}}(x-0) \Rightarrow \boxed{y = \frac{1}{2\sqrt{2}}x + \sqrt{2}}$$

(5pts.) c) Find the equation of the line l, passing through $(0, \sqrt{2})$ and perpendicular to the tangent line found in part b).

$$\text{slope of } l = -\frac{1}{\frac{1}{2\sqrt{2}}} = \boxed{-2\sqrt{2}}$$

$$y - \sqrt{2} = -2\sqrt{2}(x-0) \Rightarrow \boxed{y = (-2\sqrt{2})x + \sqrt{2}}$$

6. Evaluate the following limits (if they exist). In case the result is D.N.E, indicate whether the answer is ∞ or $-\infty$.

(6 pts.) **a)** $\lim_{x\to 3^-}\left(\frac{-7}{x-3}\right) = \frac{-7}{0^-} = -(-\infty) = \boxed{\infty}$

$x \to 3^-$
$x < 3$
$x - 3 < 0$

(6 pts.) **b)** $\lim_{x\to 2^+}\left(2 - \frac{1}{x-2}\right) = 2 - \frac{1}{0^+} = 2 - \infty = \boxed{-\infty}$

$x \to 2^+$
$x > 2$
$x - 2 > 0$

(6 pts.) **c)** $\lim_{t\to 2}\frac{t^2 - t - 2}{t^2 + 3t - 10} = \lim_{\substack{t\to 2\\ t\neq 2}}\left[\frac{(t-2)(t+1)}{(t-2)(t+5)}\right] = \lim_{t\to 2}\left(\frac{t+1}{t+5}\right) = \boxed{\frac{3}{7}}$

7. (7 pts.) **a)** If $y = y(x)$ is given implicitly by $e^x + e^y = x^2 + y^2$, use **implicit differentiation to find** $y'(x)$.

$$e^x + e^y \cdot y' = 2x + 2y \cdot y' \Rightarrow y'[e^y - 2y] = 2x - e^x$$

$$\Rightarrow \boxed{y'(x) = \frac{2x - e^x}{e^y - 2y}} \left(\text{or} = \frac{e^x - 2x}{2y - e^y}\right)$$

(8 pts.) **b)** Use **logarithmic differentiation** to find $y'(x)$ if $y(x) = \sqrt{\frac{(x+3)(x-2)}{2x+1}}$.

$$\ln y(x) = \ln\left(\frac{(x+3)(x-2)}{(2x+1)}\right)^{1/2} = \frac{1}{2}\left[\ln(x+3) + \ln(x-2) - \ln(2x+1)\right]$$

differentiate both sides w.r.t. x $\Rightarrow$

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{1}{x+3} + \frac{1}{x-2} - \frac{2}{2x+1}\right]$$

$$\Rightarrow \boxed{y' = \sqrt{\frac{(x+3)(x-2)}{2x+1}} \cdot \left[\frac{1}{2}\left(\frac{1}{x+3} - \frac{1}{x-2} - \frac{2}{2x+1}\right)\right]}$$

(5 pts.) **c)** Given that $y(x) = x\, e^{x^3}$, **evaluate** $y''(1) = ?$

$$y'(x) = 1 \cdot e^{x^3} + x \cdot \left(e^{x^3} \cdot 3x^2\right) = e^{x^3}(1 + 3x^3)$$

$$y''(x) = \left(e^{x^3} \cdot 3x^2\right)(1 + 3x^3) + e^{x^3} \cdot (9x^2)$$

$$= e^{x^3}\left[12x^2 + 9x^5\right]$$

$$y''(1) = e^1(12 + 9) = \boxed{21e}$$