ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2019-2020 Fall

FIRST MIDTERM EXAMINATION

15.11.2019

ANSWER KEY

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

DURATION: 80 minutes

Question	Grade	Out of
1		40
2		21
3		15
4		24
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number, name and signature above.

2) Check that the exam paper contains 4 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

4) Please <u>TURN OFF</u> your cellphones.

5) Calculators are <u>NOT ALLOWED</u>.

6) It is not allowed to leave the exam during the first 30 minutes.

1) (8 points each) <u>Solve</u> the following equalities and inequalities for the unknown variable x. Express the solution sets clearly.

a) $\left|1 - \frac{7x}{3}\right| + 2x = 3$

$$1-\frac{7x}{3} = 3-2x \qquad\qquad -1+\frac{7x}{3} = 3-2x$$

$$2x-\frac{7x}{3} = 3-1 \qquad\qquad 2x+\frac{7x}{3} = 3+1$$

$$\frac{-x}{3} = 2 \qquad\qquad \frac{13x}{3} = 4$$

$$\boxed{x=-6} \qquad\qquad \boxed{x=\frac{12}{13}}$$

$$S.S: \left\{-6, \frac{12}{13}\right\} //$$

b) $\left|6x - 15\right| \geq 3$

$$6x-15 \geq 3 \qquad\qquad 6x-15 \leq -3$$

$$6x \geq 18 \qquad\qquad 6x \leq 12$$

$$\boxed{x \geq 3} \qquad\qquad \boxed{x \leq 2}$$

$$S.S: (-\infty, 2] \cup [3, \infty) //$$

c) $\ln e^{2x} + e^{\ln(x^2)} + \ln e^2 = 5$

$$2x + x^2 + 2 = 5$$

$$x^2+2x+2-5 = 0$$

$$x^2+2x-3 = 0$$

$$(x+3)(x-1) = 0 \Rightarrow \cancel{\boxed{x=-3}}\ \boxed{x=1} \qquad S.S: \{1\} //$$

d) $\log_5(\sqrt{x^2+1}) + \log_5 x = 1$

$$\log_5\left(x\sqrt{x^2+1}\right) = 1 \Rightarrow x\sqrt{x^2+1} = 5$$

$$x^2(x^2+1) = 25$$

$$x^4+x^2-25 = 0$$

$$z^2+z-25 = 0$$

Two roots

$$\Delta = 1-4(-25) = 101 > 0 \Rightarrow \cancel{z_1 = \frac{-1-\sqrt{101}}{2} < 0}$$

$$z_2 = \frac{-1+\sqrt{101}}{2} > 0$$

$z_1 = x_1^2$ can not be negative

$$\text{So } x_2^2 = z_2 = \frac{-1+\sqrt{101}}{2} \Rightarrow x_2 = \mp\sqrt{\frac{-1+\sqrt{101}}{2}}$$

In which x_2 can not be negative because of logarithm.

So $S.S: \left\{\sqrt{\frac{-1+\sqrt{101}}{2}}\right\}$

e) $10^{\ln(\frac{1}{x})+2\ln(2x-1)} = 1$

$$10^{\ln\left(\frac{(2x-1)^2}{x}\right)} = 1$$

$$\Rightarrow \ln\left(\frac{(2x-1)^2}{x}\right) = 0$$

$$\Rightarrow \frac{(2x-1)^2}{x} = 1 \ (= e^0) \Rightarrow 4x^2-4x+1 = x$$

$$4x^2-5x+1 = 0 \Rightarrow (4x-1)(x-1) = 0 \Rightarrow x=\frac{1}{4},\ x=1 \qquad S.S: \{1\} //$$

! makes log negative

2) (7 points each) Given the quadratic function $y = f(x) = -x^2 + 5x + 14$,

a) Find the vertex and intercepts of its graph,

Intercepts:

$y=0 \Rightarrow -x^2+5x+14=0$

$x^2-5x-14=0$

$(x-7)(x+2)=0$

$x=7$ & $x=-2 \Rightarrow$ $\boxed{(-2,0) \& (7,0) \text{ are x-intercepts}}$

$x=0 \Rightarrow y=14$

so, $\boxed{(0,14) \text{ is y-intercept.}}$

Vertex:

$V\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

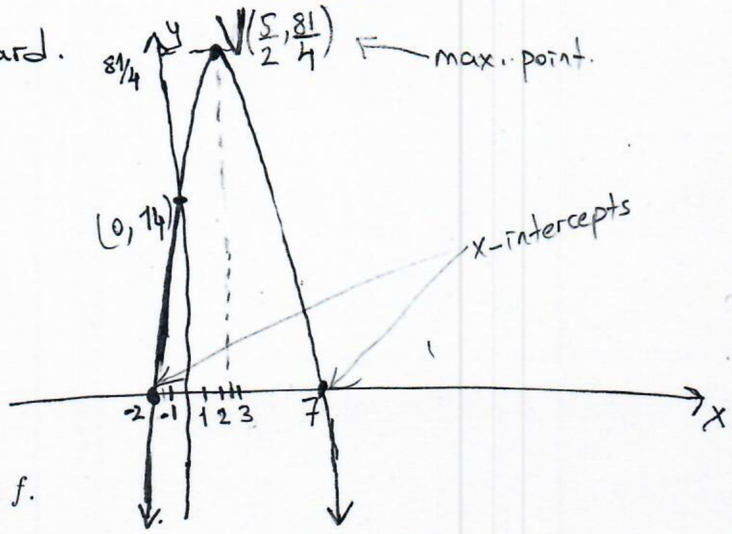
$V\left(\frac{-5}{-2}, f\left(\frac{-5}{-2}\right)\right)$

$\boxed{V\left(\frac{5}{2}, \frac{81}{4}\right)}$ vertex point.

b) Sketch the graph,

$a=-1<0$ so parabola is downward.

with the info in part (a);



c) Find the Domain and Range of the function f.

$D(f): \mathbb{R}$

$R(f): \left(-\infty, \frac{81}{4}\right]$

3) (15 points) Find the equation of the line passing through the point $P(2,5)$ and perpendicular to the line $2y + 4x - 8 = 0$.

$L_1: 2y+4x-8=0 \Rightarrow y=-2x+4$ so $\boxed{m_1=-2}$ $\Rightarrow$ Since $\boxed{m_1 \cdot m_2 = -1}$ $\Rightarrow$ $\boxed{m_2 = \frac{1}{2}}$ //

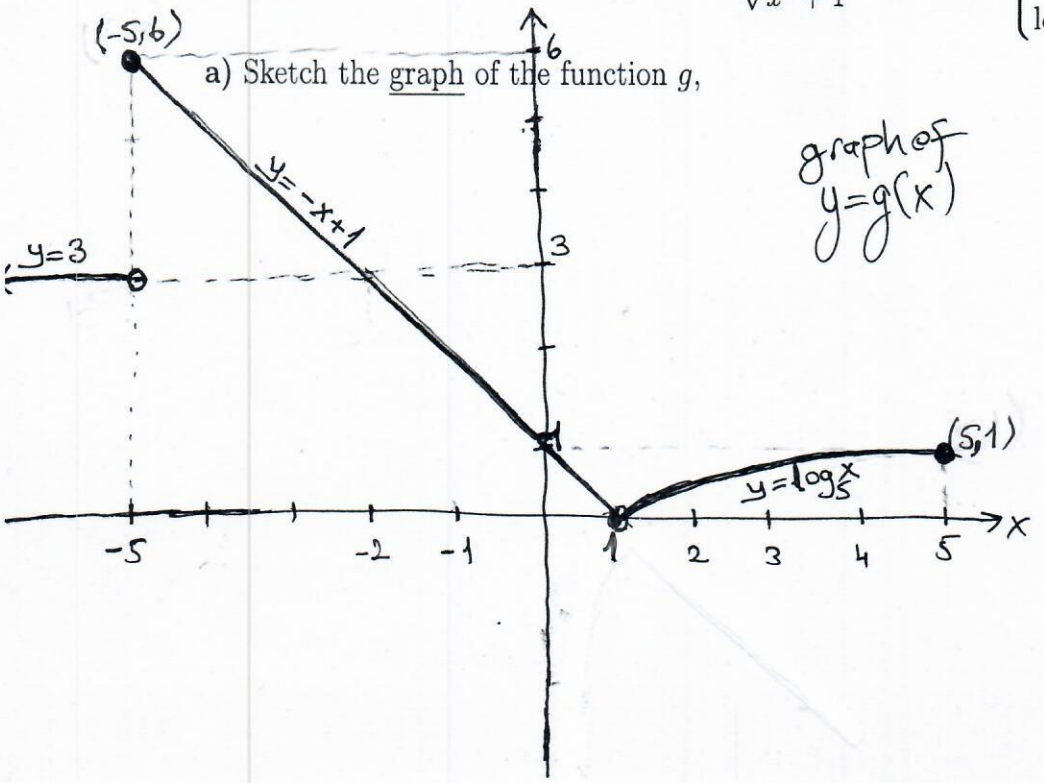
Eqn. of L_2: $S(2,5)$ & $m_2=\frac{1}{2}$ with (x_1, y_1)

$y-y_1 = m_2(x-x_1) \Rightarrow y-5=\frac{1}{2}(x-2)$

$\boxed{y=\frac{x}{2}+4}$ // is the eqn. of the line perpendicular to $2y+4x-8=0$ and passing $P(2,5)$

4) (24 points) For the given functions $f(x) = \dfrac{2}{\sqrt{x^2+1}}$ and $g(x) = \begin{cases} 3, & \text{if } x < -5 \\ -x+1, & \text{if } -5 \le x < 1 \\ \log_5 x, & \text{if } 1 < x \le 5 \end{cases}$,

a) Sketch the graph of the function g,



b) Find the Domain and Range of the each function f and g,

$D(g): (-\infty, 5]$	$R(g): [0, 6]$
$D(f): \mathbb{R}$ $R(f): (0, \infty)$	

$\longrightarrow$ For f to be defined $x^2+1 \geq 0$ & $(x^2+1 \neq 0)$

Since $x^2 \geq 0\ \forall x \in \mathbb{R}$ then $x^2+1 > 0$ is always satisfied

c) Calculate (if possible) $(f \circ g)(0)$ and $(g \circ f)(0)$.

$$(f \circ g)(0) = f(g(0)) = f(1) = \frac{2}{\sqrt{1^2+1}} = \frac{2}{\sqrt{2}} = \boxed{\sqrt{2}} \ //$$

$$\left(g(0) = -0+1 = 1 \quad (g(x) = (-x+1)) \right)$$

$$(g \circ f)(0) = g(f(0)) = g(2) = \boxed{\log_5 2} \ //$$

$$\left(f(0) = \frac{2}{\sqrt{0+1}} = 2 \right)$$