ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Spring

SECOND MIDTERM EXAMINATION

24.04.2019, 17:30

-SOLUTIONS-

Question	Grade	Out of
1		24
2		21
3		18
4		24
5		18
Total		105

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Solve the following equalities. Clearly indicate your answer.

(6 points) **a)** $e^{1-x} - 2 = 3$

$e^{1-x} = 3+2=5 \Rightarrow \underbrace{\ln e^{1-x}}_{(1-x)\underbrace{\ln e}_{1}} = \ln 5 \Rightarrow 1-x = \ln 5$

$\boxed{x = 1-\ln 5}$

(6 points) **b)** $\log_5 40 - \log_5 2 = \log_5 100 - x$

$x = \log_5 100 + \log_5 2 - \log_5 40 = \log_5\left(\frac{100\cdot 2}{40}\right) = \underbrace{\log_5 5}_{1}$

$\Rightarrow \boxed{x=1}$

(6 points) **c)** $\ln(x+6) - \ln 2 = 2\ln x$

$\underbrace{x+6>0}\Rightarrow x>-6$, $x>0$ $\Big\}\, x>0$

$\ln\left(\frac{x+6}{2}\right) = \ln x^2 \Rightarrow x^2 = \frac{x+6}{2} \Rightarrow 2x^2-x-6=0$

$(2x+3)(x-2)=0$

$\boxed{x=-\frac{3}{2},\ x=2}$

$\Rightarrow$ Since $x>0$ for $\ln x$ to be defined,

$x=-\frac{3}{2}$ can not be a solution $\Rightarrow$ So soln. is: $\boxed{x=2}$

(6 points) **d)** $\log_x(6-5x) = 2$

$x>0$
$x\neq 1$

$\Rightarrow x^2 = 6-5x \Rightarrow x^2+5x-6=0$

$(x+6)(x-1)=0$

$\boxed{x=-6 \text{ or } x=1}$

But since $\overbrace{x>0 \text{ and } x\neq 1}$ should be satisfied for $\log_x(6-5x)$ to be defined, neither $x=-6$ nor $x=1$ can be solutions.

So, <u>soln. set</u>: $\{\emptyset\}$ (<u>No solution</u> !)

2. (7 points) **a)** If $f(x) = \begin{cases} x+1, & \text{when } x \geq 1 \\ x-1, & \text{when } x < 1 \end{cases}$, then what is $\lim_{x\to 1} f(x)$? Explain your answer.

$$\lim_{\substack{x\to 1^- \\ (x<1)}} f(x) = \lim_{x\to 1^-}(x-1) = 1-1 = 0$$

$$\lim_{\substack{x\to 1^+ \\ (x>1)}} f(x) = \lim_{x\to 1^+}(x+1) = 1+1 = 2$$

since right and left limits are not equal at $x=1 \Rightarrow$

$$\boxed{\lim_{x\to 1} f(x) = \text{D.N.E.}}$$

(6 points) **b)** Find: $\lim_{x\to\infty} \dfrac{2x^2-4x+9}{8-3x^2} = ?$ (Explain your answer).

Since deg. numerator = deg. denominator $\Rightarrow$

$$\lim_{x\to\infty} \frac{2x^2-4x+9}{8-3x^2} = \text{ratio of leading term coefficients} = \frac{2}{-3} = -\frac{2}{3}$$

(8 points) **c)** For what values of the constants **a** and **b**, the function

$$f(x) = \begin{cases} \mathbf{a}x, & \text{if } x < -1 \text{ or } x > 3 \\ x^3 + \mathbf{b}, & \text{if } -1 \leq x \leq 3 \end{cases}$$

continuous for all x values?

ax & x^3+b are cont. everywhere, so we should only check the jump points:

at $x=-1$:

$$\lim_{\substack{x\to -1^- \\ (x<-1)}} f(x) = \lim_{x\to -1^-}(ax) = \boxed{-a}$$

$$\lim_{\substack{x\to -1^+ \\ (x>-1)}} f(x) = \lim_{x\to -1^+}(x^3+b) = (-1)^3+b = \boxed{-1+b}$$

For continuity at $x=-1$: $\boxed{-a=-1+b}$ $(a=1-b)$

at $x=3$:

$$\lim_{\substack{x\to 3^- \\ (x<3)}} f(x) = \lim_{x\to 3^-}(x^3+b) = (3)^3+b = \boxed{27+b}$$

$$\lim_{\substack{x\to 3^+ \\ (x>3)}} f(x) = \lim_{x\to 3^+}(ax) = \boxed{3a}$$

For continuity at $x=3$: $\boxed{27+b=3a}$

$$\left.\begin{array}{l} a=1-b \\ 27+b=3a \end{array}\right\} 27+b=3(1-b) \Rightarrow 4b=-24 \quad \boxed{b=-6}$$

$$\Rightarrow a=1-(-6)=7 \quad \boxed{a=7}$$

3. (9 points) **a)** Find the slope of the curve $y = \dfrac{2x+5}{x-3}$ at the point $(4, 13)$.

$$\text{slope} = m = y'(4) \Rightarrow y'(x) = \frac{(2)(x-3)-(1)(2x+5)}{(x-3)^2}$$

$$\Rightarrow y'(x) = \frac{\cancel{2x}-6-\cancel{2x}-5}{(x-3)^2} = \frac{-11}{(x-3)^2} \Rightarrow y'(4) = \frac{-11}{(4-3)^2} = -11$$

$$\Rightarrow \text{slope} = y'(4) = -11$$

(9 points) **b)** Find the equation of the tangent line to the curve $y = \ln(x+3)$ when $x = -2$.

$$x=-2 \Rightarrow y(-2) = \ln(-2+3) = \ln 1 = 0 \Rightarrow \text{pt.}: (-2,0)$$

$$\text{equation of tg. line: } y-0 = m(x-(-2)), \quad m = y'(-2)$$

$$y'(x) = \frac{1}{x+3} \Rightarrow y'(-2) = \frac{1}{-2+3} = 1 = m$$

$$\Rightarrow \text{eqn. of tg. line: } y-0 = 1(x+2)$$

$$\Rightarrow \boxed{y = x+2}$$

4. Evaluate the derivatives of the following functions. Simplify your answers as much as possible.

(8 points) **a)** $f(x) = \log_5(4x + x^2 e^{3x}) = \dfrac{\ln(4x+x^2e^{3x})}{\ln 5}$

$$\Rightarrow f'(x) = \frac{1}{4x+x^2e^{3x}} \cdot \left(4 + 2xe^{3x} + x^2(3e^{3x})\right) \cdot \frac{1}{\ln 5}$$

$$= \boxed{\frac{4+2xe^{3x}+3x^2e^{3x}}{(4x+x^2e^{3x})(\ln 5)}}$$

(8 points) **b)** $y(x) = \sqrt{x}\ln x$

$$y'(x) = \left(\frac{1}{2\sqrt{x}}\right)(\ln x) + (\sqrt{x})\left(\frac{1}{x}\right) = \frac{\ln x}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$$

$$= \boxed{\frac{1}{\sqrt{x}}\left(\frac{\ln x}{2} + 1\right)}$$

(8 points) **c)** $g(x) = e^{(3x^4+1)^2}$

$$g'(x) = e^{(3x^4+1)^2} \cdot \left(2(3x^4+1)\right)(12x^3)$$

$$= (24x^3)(3x^4+1)\left(e^{(3x^4+1)^2}\right)$$

$$= \boxed{(72x^7 + 24x^3)\left(e^{(3x^4+1)^2}\right)}$$

5. Given that $f(1) = 0$, $f'(1) = 3$, $f(2) = -1$, $f'(2) = -2$, $g(1) = 4$, $g'(1) = 2$, $g(2) = 1$ and $g'(2) = 5$;

(9 points) **a)** If $h(x) = \dfrac{2 + f(x)}{g(x) - x^2 + 1}$, then compute $h'(1)$.

$$h'(x) = \frac{(f'(x))(g(x) - x^2 + 1) - (g'(x) - 2x)(2 + f(x))}{(g(x) - x^2 + 1)^2}$$

$$h'(1) = \frac{(f'(1))(g(1) - \cancel{1^2} + \cancel{1}) - (g'(1) - 2(1))(2 + f(1))}{(g(1) - \cancel{1^2} + \cancel{1})^2}$$

$$= \frac{(3)(4) - \cancel{(2-2)}}{(4)^2} = \frac{12}{16} = \boxed{\frac{3}{4}}$$

(9 points) **b)** If $k(x) = f(g(2x))$, then compute $k'(1)$.

$$k'(x) = f'(g(2x)) \cdot g'(2x) \cdot 2$$

$$k'(1) = f'(g(2)) \cdot g'(2) \cdot 2$$

$$= f'(1) \cdot 5 \cdot 2$$

$$= 3 \cdot 5 \cdot 2 = \boxed{30}$$