ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Spring

FIRST MIDTERM EXAMINATION

18.03.2019, 17:30

-SOLUTIONS-

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

Question	Grade	Out of
1		22
2		20
3		10
4		15
5		23
6		15
Total		105

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions. Express the solutions clearly.

(5 points) **a)** $(2x+1)^2+3x=-2$

$(4x^2+4x+1)+3x+2=0$

$4x^2+7x+3=0$

$(4x+3)(x+1)=0$

$x_1=-\frac{3}{4}$ $x_2=-1$

Soln.: $\{-\frac{3}{4}, -1\}$

(5 points) **b)** $\frac{2}{x^2-16}=\frac{1}{x-4}+\frac{1}{x+4}$

$x\neq 4$, $x\neq -4$

$\frac{2}{x^2-16}=\frac{(x+4)+(x-4)}{(x-4)(x+4)}=\frac{2x}{x^2-16} \Rightarrow 2=2x$

$x=1$

Soln.: $\{1\}$

(6 points) **c)** $\sqrt{x^2-x-5}-2x=3$

$\sqrt{x^2-x-5}=3+2x \Rightarrow \left(\sqrt{x^2-x-5}\right)^2=(3+2x)^2 \Rightarrow x^2-x-5=9+12x+4x^2$

$\Rightarrow 3x^2+13x+14=0$

$(3x+7)(x+2)=0$ → $x_1=-\frac{7}{3}$, $x_2=-2$

$x_1=-\frac{7}{3} \Rightarrow$ into the eqn.: $\sqrt{(-\frac{7}{3})^2-(-\frac{7}{3})-5}=\frac{5}{3}\neq -\frac{5}{3}=3+2(-\frac{7}{3}) \Rightarrow x_1=-\frac{7}{3}$ doesn't satisfy the eqn. $\Rightarrow$ not a soln.

$x_2=-2 \Rightarrow$ into the eqn.: $\sqrt{(-2)^2-(-2)-5}=1\neq -1=3+2(-2)$

$\Rightarrow x_2=-2$ doesn't satisfy the eqn. $\Rightarrow$ not a soln.

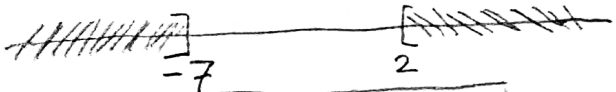
$\Rightarrow$ **No Solution**

(6 points) **d)** $|2x+5|+1\geq 10$

$|2x+5|\geq 9 \Rightarrow 2x+5\leq -9$ or $2x+5\geq 9$

$2x\leq -14$ $\quad 2x\geq 4$

$x\leq -7$ or $x\geq 2$



Soln.: $(-\infty, -7]\cup[2, \infty)$

2. Let $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x-1}$.

(2 points) **a)** Find $(f-g)(4)$.

$$(f-g)(x) = \frac{1}{x-2} - \sqrt{x-1} \Rightarrow (f-g)(4) = \frac{1}{4-2} - \sqrt{4-1} = \boxed{\frac{1}{2} - \sqrt{3}}$$

(3 points) **b)** Find $(f \circ g)(x)$.

$$f(g(x)) = f\left(\sqrt{x-1}\right) = \boxed{\frac{1}{\sqrt{x-1}-2}}$$

(3 points) **c)** Find $(g \circ f)(x)$.

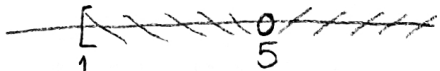
$$g(f(x)) = g\left(\frac{1}{x-2}\right) = \sqrt{\frac{1}{x-2} - 1} = \sqrt{\frac{1-x+2}{x-2}} = \boxed{\sqrt{\frac{3-x}{x-2}}}$$

(4 points) **d)** Find Domain(g).

$$x-1 \geq 0 \Rightarrow x \geq 1 \Rightarrow \boxed{[1, \infty)}$$

(4 points) **e)** Find Domain($f \circ g$).

$$\left.\begin{array}{l} \sqrt{x-1} \neq 2 \Rightarrow x-1 \neq 4 \Rightarrow x \neq 5 \\ x-1 \geq 0 \Rightarrow x \geq 1 \end{array}\right\}$$



$\boxed{[1,5) \cup (5,\infty)}$ (or $[1,\infty) \setminus \{5\}$)

(4 points) **f)** Find Domain($g \circ f$).

$$\frac{3-x}{x-2} \geq 0$$

$$x \neq 2$$

	$-\infty$		2		3		∞
$x-2$	$-$	$-$	0	$+$	$+$	$+$	
$3-x$	$+$	$+$	$+$	$+$	0	$-$	
$\frac{3-x}{x-2}$	$-$	$+$	∄	$+$	0	$-$	

Domain: $\boxed{(2,3]}$

3. (8 points) **a)** Use the graph of the function $f(x) = -|x|$ and transformation techniques to sketch the graph of the function $g(x) = 3 - |x - 2|$. Clearly indicate the vertex, x and y-intercepts (if any) on the graph of $g(x)$.

$g(x) = 3 - |x-2|$

vertex: $(2,3)$

x-intercept(s): $(-1,0)$ & $(5,0)$

$y = 0 \Rightarrow |x-2| = 3$

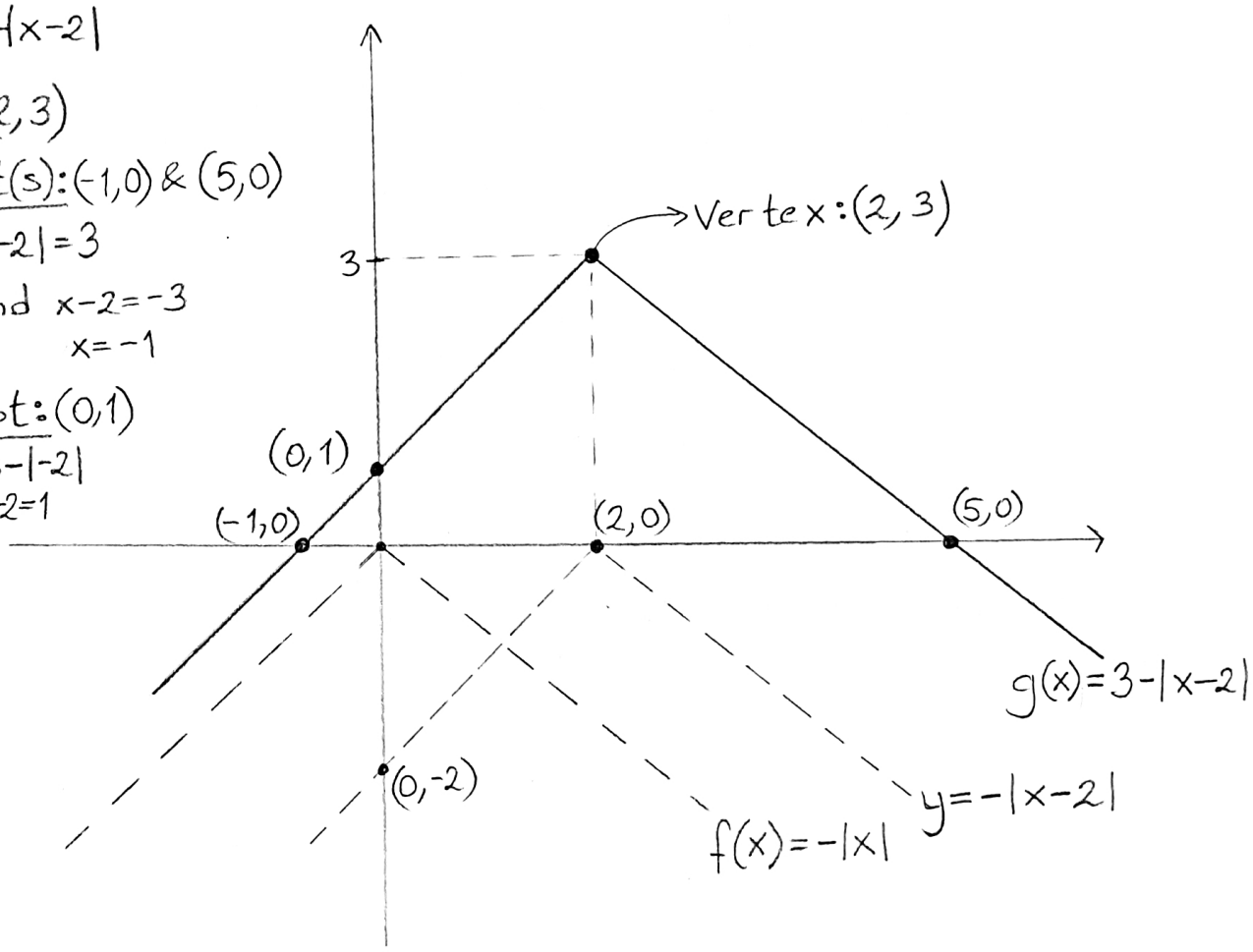
$x - 2 = 3$ and $x - 2 = -3$

$x = 5$ $\quad$ $x = -1$

y-intercept: $(0,1)$

$x = 0 \Rightarrow y = 3 - |-2|$

$y = 3 - 2 = 1$



(2 points) **b)** Write the Range of $g(x)$. $\boxed{(-\infty, 3]}$

4. Consider the function $f(x) = -(x-2)^2 + 1$.

(6 points) **a)** Find the vertex, x-intercept(s) and y-intercept(s) (if any) of $f(x)$.

Vertex: $(2,1)$

Intercepts: $x=0 \Rightarrow y=-4+1=-3 \Rightarrow \boxed{(0,-3)\ y\text{-intercept}}$

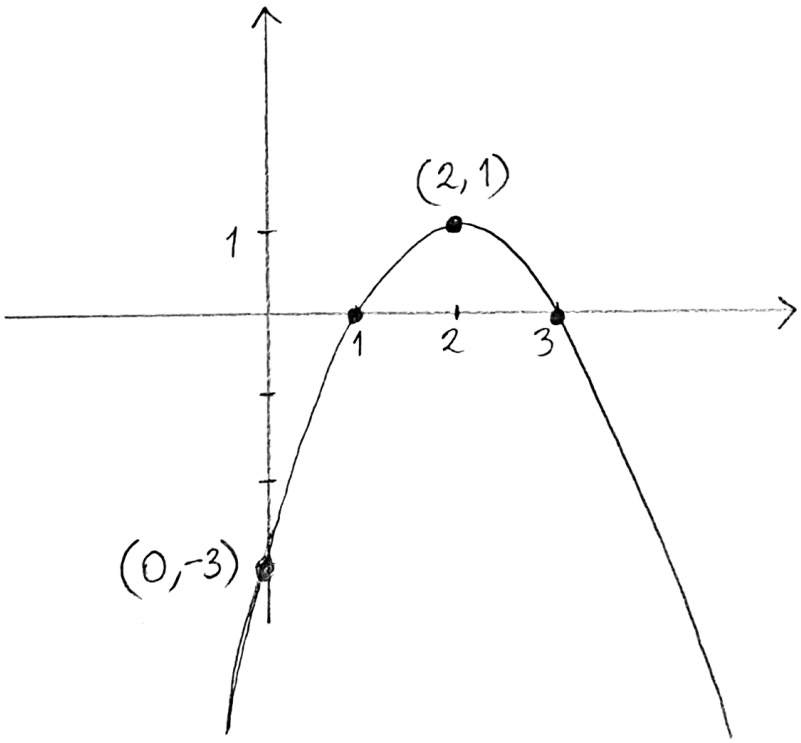
$y=0 \Rightarrow (x-2)^2=1 \Rightarrow x-2=\mp 1 \Rightarrow x=2+1$ or $x=2-1$

$\Downarrow$ $\Downarrow$

$x=3$ $\quad x=1$

x-intercepts: $\boxed{(3,0),(1,0)}$

(5 points) **b)** Sketch the graph of f.



(4 points) **c)** Find the Domain and Range of $f(x)$.

Domain: $\mathbb{R}$

Range: $(-\infty, 1]$

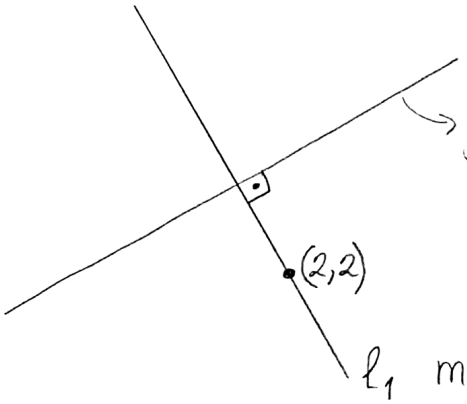
5. (7 points) **a)** Find the equation of the line passing through the points $(-2,1)$ and $(-1,3)$.

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{3-1}{-1-(-2)}=\frac{2}{1}=2 \Rightarrow y-y_1=m(x-x_1)$$

$$y-1=2(x-(-2))=2x+4$$

$$\boxed{y=2x+5}$$

(8 points) **b)** Find the equation of the line l_1 passing through the point $(2,2)$ and perpendicular to the line $3y-6x+9=0$.



$3y-6x+9=0 \Rightarrow 3y=6x-9 \Rightarrow y=2x-3$

$y=2x-3$ $(m=2)$

l_1 m_1: slope of $l_1=\frac{-1}{m}=-\frac{1}{2}$

l_1 equation:

$y-2=(-\frac{1}{2})(x-2)$

$y=-\frac{1}{2}x+1+2$

$$\boxed{y=-\frac{1}{2}x+3}$$

(8 points) **c)** Sketch the graph of the line given by the equation $\frac{x}{2}-\frac{y}{3}=5$.

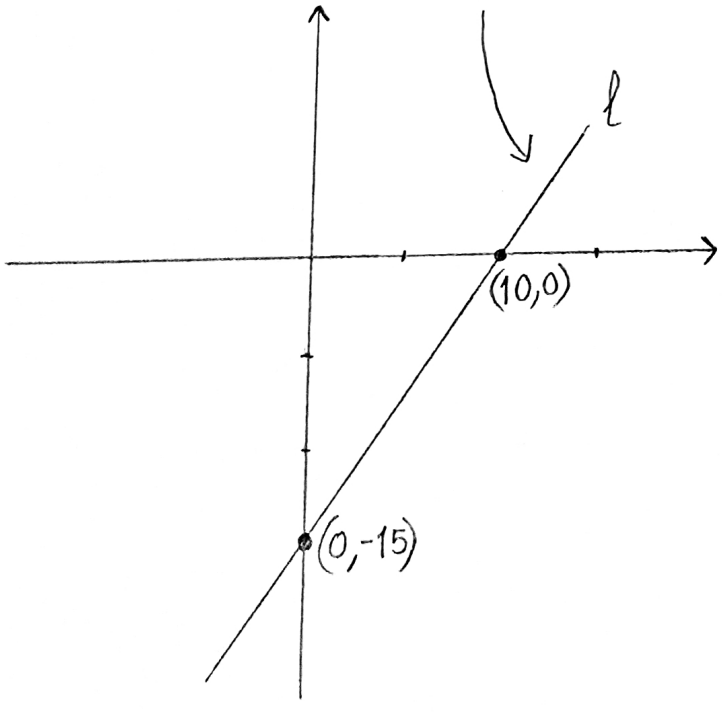
$\frac{x}{2}-\frac{y}{3}=5 \Rightarrow 3x-2y=30 \Rightarrow$

$x=0 \Rightarrow -2y=30 \Rightarrow y=-15$

$(0,-15) \rightarrow$ y-intercept

$y=0 \Rightarrow 3x=30 \Rightarrow x=10$

$(10,0) \rightarrow$ x-intercept



6. (6 points) **a)** Find the inverse function, $f^{-1}(x)$, of $f(x) = 3 - \frac{1}{x}$.

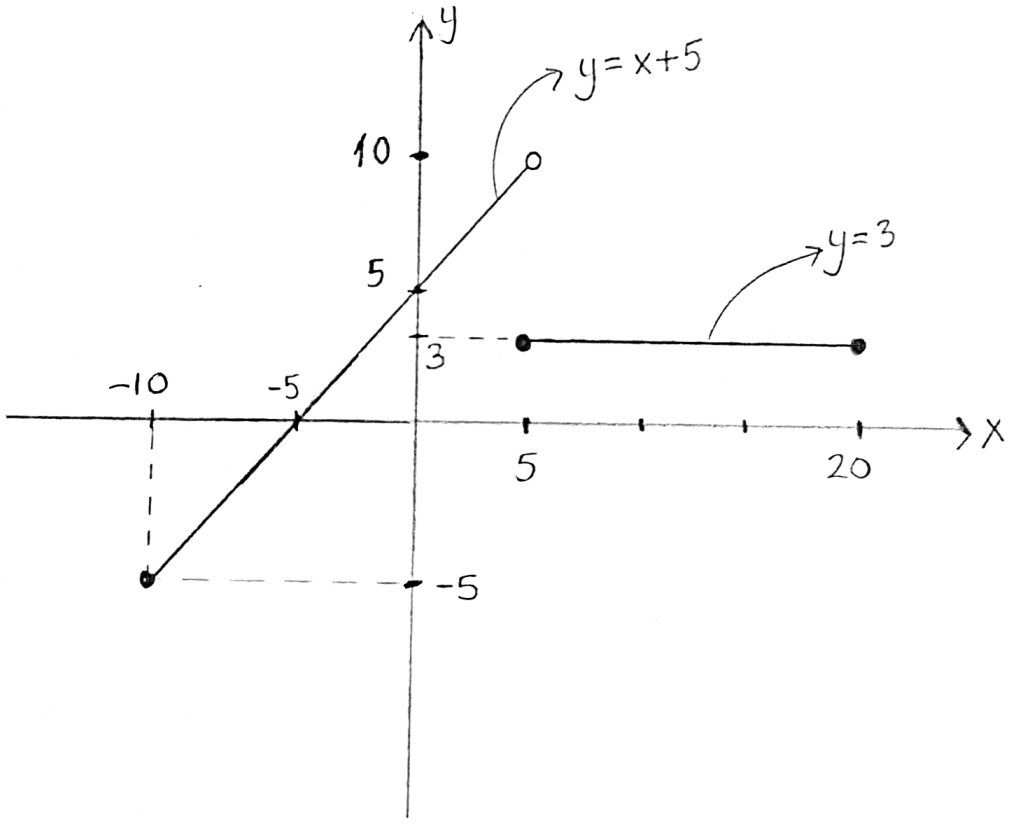
$$y = 3 - \frac{1}{x} \Rightarrow \frac{1}{x} = 3 - y \Rightarrow x = \frac{1}{3-y} \Rightarrow y = \frac{1}{3-x}$$

$$x \leftrightarrow y$$

Let $y = f^{-1}(x) \Rightarrow \boxed{f^{-1}(x) = \frac{1}{3-x}}$

b) For the piece-wise defined function $f(x) = \begin{cases} x+5, & \text{if } -10 \le x < 5 \\ 3, & \text{if } 5 \le x \le 20 \end{cases}$

(6 points) **i)** Sketch the graph of $f(x)$, clearly indicating the important points.



(3 points) **ii)** Find the Domain and Range of $f(x)$.

Domain: $[-10, 20]$

Range: $[-5, 10)$