ÇANKAYA UNIVERSITY

Department of Mathematics

SOLUTIONS

MATH 105 - Business Mathematics I

2018-2019 Fall

SECOND MIDTERM EXAMINATION

11.12.2018, 17:30

Question	Grade	Out of
1		17
2		20
3		15
4		17
5		18
6		18
Total		105

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

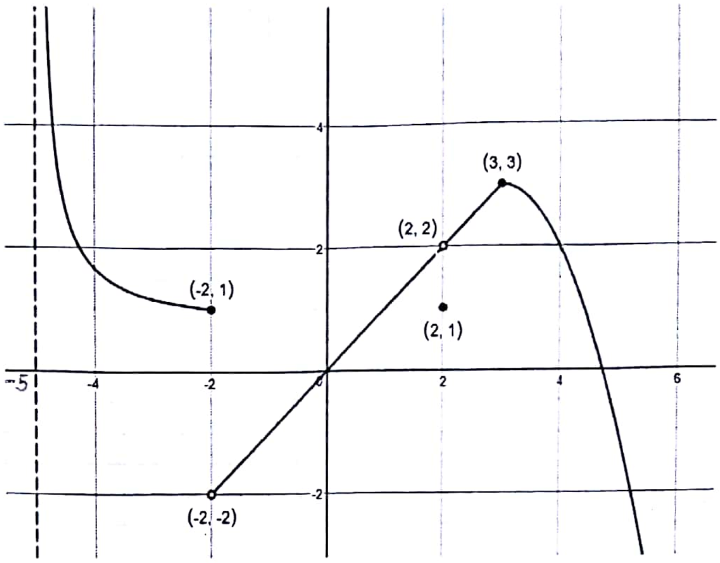
IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Consider the following graph of the function $f(x)$.



Determine the indicated limits if they exist. Explain your answers in detail.

a) (3 points) $\lim_{x\to -5^+} f(x) = +\infty$ (DNE)

b) (4 points) $\lim_{x\to -2} f(x) =$ D.N.E.

$\lim_{x\to -2^-} f(x) = 1$

$\lim_{x\to -2^+} f(x) = -2$

$\neq \Rightarrow \lim_{x\to -2} f(x) =$ D.N.E.

c) (4 points) $\lim_{x\to 3} f(x) = 3$

d) (6 points) Is $f(x)$ continuous at $x = 2$. Explain why or why not.

$f(2) = 1$

$\lim_{x\to 2} f(x) = 2$

$\neq \Rightarrow f$ is <u>not</u> continuous at $x=2$

$\left(\lim_{x\to 2^-} f(x) = 2 = \lim_{x\to 2^+} f(x)\right)$

2.Evaluate the following limits if they exist. Express the solutions clearly.

a) (5 points) $\lim_{z\to 0} \dfrac{z^2-5z-4}{z^2+1} = \dfrac{-4}{1} = \boxed{-4}$

b) (5 points) $\lim_{x\to 2} \dfrac{3x^2-x-10}{x^2+5x-14} = \dfrac{3(4)-2-10}{4+10-14} = \dfrac{0}{0}$

$$= \lim_{\substack{x\to 2 \\ (x\neq 2)}} \frac{(3x+5)\cancel{(x-2)}}{(x+7)\cancel{(x-2)}} = \frac{3(2)+5}{2+7} = \boxed{\frac{11}{9}}$$

c) (5 points) $\lim_{x\to 4} \dfrac{x-4}{4-\sqrt{x+12}} = \dfrac{0}{0}$

$$= \lim_{x\to 4} \frac{(x-4)(4+\sqrt{x+12})}{(4-\sqrt{x+12})(4+\sqrt{x+12})} = \lim_{x\to 4} \frac{-\cancel{(4-x)}(4+\sqrt{x+12})}{\underbrace{16-(x+12)}_{\cancel{(4-x)}}}$$

$$= -(4+\sqrt{4+12}) = \boxed{-8}$$

d) (5 points) $\lim_{x\to\infty} \dfrac{3-4x-2x^3}{5x^3-8x+1} = \boxed{\dfrac{-2}{5}}$

$\deg p(x) = \deg q(x) = 3$

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3. (15 points) Find the points of discontinuity (if any) of the function

$$f(x) = \begin{cases} e^x + 3 - x^2, & \text{if } x < 0, \\ 3x - 2, & \text{if } 0 \le x < 1, \\ 2 - x^2 + \ln x, & \text{if } 1 \le x. \end{cases}$$

Explain your answer in detail.

at x=0: $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (e^x + 3 - x^2) = \underbrace{e^0}_{=1} + 3 - 0 = 4$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3x - 2) = -2$

$\Rightarrow \lim_{x \to 0} f(x) = \text{D.N.E.}$

$\Rightarrow$ f is not continuous at $x=0$

at x=1: $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3x-2) = 3-2 = 1$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (2 - x^2 + \ln x) = 2 - 1 + 0 = 1$

$\Rightarrow \lim_{x \to 1} f(x) = 1 = f(1)$

$\Rightarrow$ f is continuous at $x=1$.

At all other pts.; $e^x + 3 - x^2$, $3x-2$, $2 - x^2 + \ln x$ $(x \geq 1)$

f is continuous $\Rightarrow$

$\boxed{\text{Only discontinuity point of } f \text{ is } x=0}$

4. Consider the function $f(x) = x^2 - 3$. $\Rightarrow f(x+h) = (x+h)^2 - 3 = x^2 + 2xh + h^2 - 3$

a) (6 points) Find $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(\cancel{x^2} + 2xh + h^2 \cancel{-3}) - (\cancel{x^2} \cancel{-3})}{h}$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(2x+h)}{\cancel{h}} = \boxed{2x}$$

b) (5 points) Calculate the slope m of the tangent line drawn to $f(x) = x^2 - 3$ at the point where $x = 2$ using $\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = f'(2) = (2x)\Big|_{x=2} = \boxed{4}$

c) (6 points) Find the equation of the tangent line described in part **b)** above.

$f(2) = 2^2 - 3 = 1 \Rightarrow$ tangent line at $(2,1)$:

$$y - 1 = 4(x-2)$$

$$\Rightarrow y = 4x - 8 + 1 = 4x - 7$$

$$\Rightarrow \boxed{y = 4x - 7}$$

$4x^{-\frac{3}{2}}$ $\quad$ $2\log_2(8x+5)$

5. a) (9 points) If $f(x) = \frac{7}{3}x^3 - 15\sqrt[5]{x^9} + \frac{4}{x^{3/2}} + 3^{(1+\sqrt{x})} + \log_2\left[(8x+5)^2\right]$ then find $f'(x)$.

$$f'(x) = \frac{7}{3}(3x^2) - 15\left(\frac{9}{5}\right)x^{\frac{9}{5}-1} - 4\left(\frac{3}{2}\right)x^{-\frac{3}{2}-1} + 3^{(1+\sqrt{x})} \cdot \frac{1}{2\sqrt{x}} \cdot \ln 3 + \frac{2}{\ln 2}\left(\frac{8}{8x+5}\right)$$

$$= \boxed{7x^2 - 27x^{\frac{4}{5}} - 6x^{-5/2} + 3^{(1+\sqrt{x})} \cdot \frac{\ln 3}{2\sqrt{x}} + \frac{16}{\ln 2}\left(\frac{1}{8x+5}\right)}$$

b) (9 points) Find y' using implicit differentiation, and simplify your answer as much as possible for $xe^y = \ln(x-y) + 5$.

$$\left(1.e^y + x.e^y \cdot y'\right) = \frac{1}{x-y}(1-y')$$

$$e^y - \frac{1}{x-y} = \left(-\frac{1}{x-y} - xe^y\right)y'$$

$$\Rightarrow y' = -\frac{\frac{e^y(x-y)-1}{\cancel{(x-y)}}}{\frac{1+x(x-y)e^y}{\cancel{(x-y)}}} = \boxed{\frac{1-e^y(x-y)}{1+e^y.x(x-y)}}$$

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6. Given that $f(1) = 0$, $f'(1) = 3$, $f(2) = -1$, $f'(2) = -2$, $g(1) = 4$, $g'(1) = 2$, $g(2) = 1$ and $g'(2) = 5$

a) (9 points) If $h(x) = \dfrac{2 + f(x)}{g(x) - x^2 + 1}$ then compute $h'(1)$.

$$h'(x) = \frac{[0+f'(x)][g(x)-x^2+1] - (g'(x)-2x)(2+f(x))}{(g(x)-x^2+1)^2}$$

$$h'(1) = \frac{[0+f'(1)][g(1)-1^2+1] - [g'(1)-2](2+f(1))}{(g(1)-1^2+1)^2}$$

$$= \frac{(3)(4-0) - \cancel{(2-2)}^{0}(2+0)}{(4-0)^2} = \frac{12-0}{16} = \boxed{\frac{3}{4}}$$

b) (9 points) If $k(x) = f(g(2x))$ then compute $k'(1)$.

$$k'(x) = f'(g(2x)) \cdot g'(2x) \cdot 2$$

$$k'(1) = f'(g(2)) \cdot g'(2) \cdot 2$$

$$= f'(1) \cdot 5 \cdot 2 = (3)(5)(2) = \boxed{30}$$