ÇANKAYA UNIVERSITY

Department of Mathematics

- SOLUTIONS -

MATH 105 - Business Mathematics I

2018-2019 Fall

FIRST MIDTERM EXAMINATION

06.11.2018, 17:30

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

Question	Grade	Out of
1		18
2		17
3		15
4		18
5		16
6		21
Total		105

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions. Clearly indicate the solution sets.

a) (9 points) $\sqrt{x+3}+1=3\sqrt{x}$

$$(\sqrt{x+3}+1)^2=(3\sqrt{x})^2 \Rightarrow x+3+2\sqrt{x+3}+1=9x$$

$$\Rightarrow 2\sqrt{x+3}=8x-4 \Rightarrow \sqrt{x+3}=4x-2 \Rightarrow x+3=16x^2-16x+4$$

$$\Rightarrow 16x^2-17x+1=0 \Rightarrow (16x-1)(x-1)=0 \Rightarrow x=\frac{1}{16} \text{ or } x=1$$

Check:

$x=1$: $\sqrt{1+3}+1=2+1=3=3\sqrt{1}$ ✓

$x=\frac{1}{16}$: $\sqrt{\frac{1}{16}+3}+1=\frac{7}{4}+1=\frac{11}{4}\neq 3\cdot\sqrt{\frac{1}{16}}$

Hence, $\boxed{\text{Soln set}=\{1\}}$

---

b) (9 points) $\left|\frac{3x-8}{2}\right|+x\geq 4$

$$\left|\frac{3x-8}{2}\right|\geq 4-x \Rightarrow |3x-8|\geq 8-2x \Rightarrow 3x-8\geq 8-2x$$

or

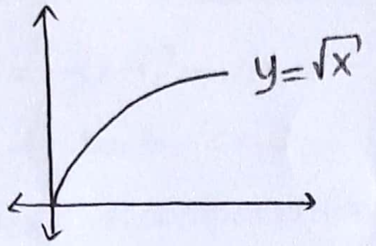
$$3x-8\leq -(8-2x)$$

$*\ 3x-8\geq 8-2x \Rightarrow 5x\geq 16 \Rightarrow x\geq \frac{16}{5}$

$*\ 3x-8\leq -(8-2x) \Rightarrow x\leq 0$

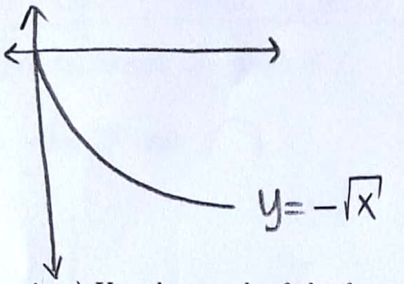
$\therefore \boxed{\text{Soln set}=(-\infty,0]\cup[\tfrac{16}{5},\infty)}$

2. **a)** (4 points) Sketch the graph of $f(x) = \sqrt{x}$



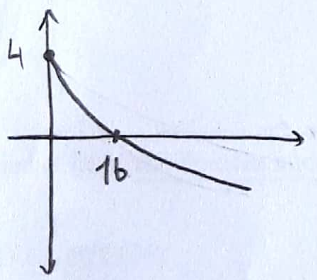
b) (4 points) Use the graph of the function $f(x) = \sqrt{x}$ obtained in part **a)** and the transformation techniques to sketch the graph of the function $g(x) = -\sqrt{x}$.

To obtain graph of $g(x)=-\sqrt{x}$, we should reflect the graph of $f(x)=\sqrt{x}$ about x-axis.



c) (5 points) Use the graph of the function $g(x) = -\sqrt{x}$ obtained in part **b)** and the transformation techniques to sketch the graph of the function $h(x) = -\sqrt{x} + 4$. Clearly indicate the intercepts (If any).

To obtain the graph of $h(x)=-\sqrt{x}+4$, we should shift the graph of $g(x)=-\sqrt{x}$ upword 4 units.



x-intercept: $y=0 \Rightarrow -\sqrt{x}+4=0$

$\Rightarrow \sqrt{x}=4 \Rightarrow x=16$

d) (4 points) Find the domain and the range of $h(x) = -\sqrt{x} + 4$.

Using graph:

Domain: $[0,\infty)$

Range: $(-\infty,4]$

3. Consider the function $f(x) = -(x+1)^2 + 8x + 1$.

a) (6 points) Find the vertex, x-intercept(s) and y-intercept(s) of $f(x)$ (If any).

$f(x) = -(x+1)^2 + 8x + 1 = -(x^2 + 2x + 1) + 8x + 1 = -x^2 + 6x$

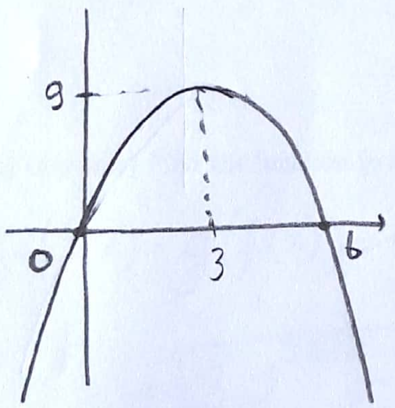
$a = -1, \; b = 6, \; c = 0.$

- Vertex: x-component $= \frac{-b}{2a} = 3.$ y-component: $y = f(3) = 9$ $\Bigg\}$ $\boxed{(3,9) \text{ is vertex}}$
- y-intercept: $x = 0 \Rightarrow y = f(0) = 0 \Rightarrow \boxed{(0,0) \text{ is y-int}}$
- x-intercept: $y = 0 \Rightarrow -x^2 + 6x = -x(x-6) = 0 \Rightarrow x = 0$ or $x = 6.$

$\Rightarrow \boxed{(0,0) \text{ and } (6,0) \text{ are x-int.}}$

b) (5 points) Sketch the graph of f.

$a = -1 < 0 \Rightarrow \cap$



c) (4 points) Find the domain and the range of $f(x)$.

Using graph:

Domain $= \mathbb{R}$

Range $= (-\infty, 9]$

4. Consider the functions $f(x) = 2^{x+1}$ and $g(x) = \sqrt{3x^2 - 12}$.

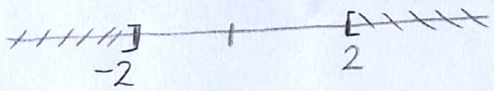
a) (3 points) Find the domain of the function $f(x)$.

$f(x) = 2^{x+1} \Rightarrow \text{Dom} f = \mathbb{R}$

b) (6 points) Find the domain of the function $g(x)$.

$\text{Dom } g = \{x \mid 3x^2 - 12 \geq 0\} = \boxed{(-\infty, -2] \cup [2, \infty)}$

$3x^2 \geq 12 \Rightarrow x^2 \geq 4 \Rightarrow x \geq 2$ or $x \leq -2$



c) (4 points) Find the function $(g \circ f)(x)$.

$(g \circ f)(x) = g(f(x)) = g(2^{x+1}) = \sqrt{3(2^{x+1})^2 - 12}$

$= \sqrt{3 \cdot 2^{2x+2} - 12} = \sqrt{3 \cdot 2^{2x} \cdot 2^2 - 12} = \boxed{\sqrt{12 \cdot 2^{2x} - 12}}$

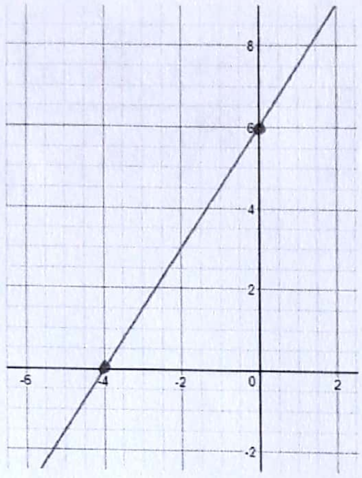
$\left(\text{or } = \sqrt{3 \cdot 4^{x+1} - 12} \right)$

d) (5 points) Find the domain of the function $(g \circ f)(x)$.

$\text{Dom of } (g \circ f)(x) = \{x \mid 12(2^{2x} - 1) \geq 0\} = \boxed{[0, \infty)}$

$\Rightarrow 2^{2x} \geq 1 = 2^0 \Rightarrow 2x \geq 0 \Rightarrow \boxed{x \geq 0}$

5. a) (6 points) Find the equation of the following line l_1 :



l_1 passing through $(0,6)$ & $(-4,0)$ $\Rightarrow$

$$\text{slope} = m_1 = \frac{6-0}{0-(-4)} = \frac{6}{4} = \frac{3}{2}$$

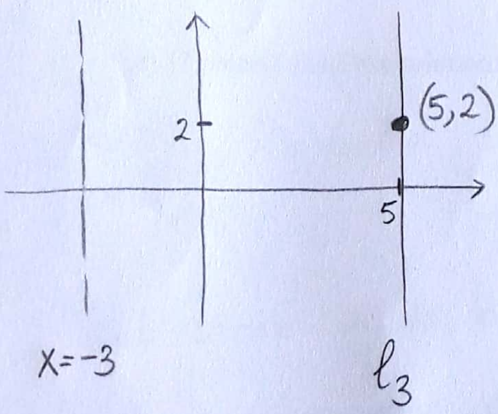
$$y-6 = \frac{3}{2}(x-0) \Rightarrow \boxed{y = \frac{3}{2}x + 6}$$

b) (6 points) Find the equation of the line l_2 passing through the point $(6,2)$ and perpendicular to the line l_1 given in part **a)**.

l_2 with slope $m_2 = \frac{-1}{m_1} = -\frac{2}{3}$

$$y-2 = -\frac{2}{3}(x-6) \Rightarrow y-2 = -\frac{2}{3}x + 4 \Rightarrow \boxed{y = -\frac{2}{3}x + 6}$$

c) (4 points) Find the equation of the line l_3 passing through the point $(5,2)$ and parallel to the line $x = -3$.



eqn. of l_3: $\boxed{x = 5}$

Scanned by CamScanner

6. (7 points) a) Simplify the expression: $\ln \sqrt[5]{\frac{(x+2)^2(x+9)^3}{(x+1)^4}}$.

$$\ln\left[\frac{(x+2)^{2/5}\cdot(x+9)^{3/5}}{(x+1)^{4/5}}\right] = \ln(x+2)^{2/5} + \ln(x+9)^{3/5} - \ln(x+1)^{4/5}$$

$$= \boxed{\frac{2}{5}\ln(x+2) + \frac{3}{5}\ln(x+9) - \frac{4}{5}\ln(x+1)}$$

b) (7 points) Find the solution set of: $3^{(2\log_3 x)}9^{(\log_4 2)} = 6$.

$$3^{2\log_3 x} = 3^{\log_3 x^2} = x^2$$

$$\log_4 2 = \frac{\log_2 2}{\log_2 4} = \frac{1}{\log_2 2^2} = \frac{1}{2\log_2 2} = \frac{1}{2}$$

$$\Rightarrow 3^{(2\log_3 x)} \cdot 9^{(\log_4 2)} = x^2 \cdot 9^{1/2} = 3x^2 = 6 \Rightarrow x^2 = 2 \Rightarrow x = \mp\sqrt{2}$$

But $x > 0 \Rightarrow$ $\boxed{\text{Solution set} = \{\sqrt{2}\}}$

c) (7 points) Find the solution set of: $\log_x(6-5x) = 2$.

$$\left.\begin{matrix} x>0 \\ x\neq 1 \end{matrix}\right\} \quad x^2 = 6-5x \Rightarrow x^2+5x-6=0 \Rightarrow (x+6)(x-1)=0$$

$$x=-6, \quad x=1$$

But since x is base and $x>0, x\neq 1$, no valid x value solves the above eqn. $\Rightarrow$ $\boxed{\text{Soln. set} = \{\emptyset\}}$

(No solution)

Scanned by CamScanner