

Logarithmic and Exponential Functions

Q1. Simplify;

a) $\frac{2^{3x+5}}{8^{x-2}}$ b) $\log 10000 - 6 \log \sqrt{10}$ c) $\frac{\log 27 + \log 20 - \log 36}{\log 40 - \log 8 + \log 3}$
d) $\log_2 1 + \log_{\frac{1}{2}} 2 + \log_2 16 + 2 \log_4 8$

Q2. Solve ;

a) $\log_x(6 - 4x - x^2) = 2$ b) $\log_2(x + 3) = -2$ c) $e^{2 \ln(2x)} = 4$
d) $\log_3(\log_9 81) = \log_{\sqrt{3}} x$ e) $\frac{\log(x^2 - \frac{1}{2}) + \log 2}{\log(x + 2)} = 2$

Q3. Solve

a) $e^{\ln(3x+4)} = 10$ b) $e^{\ln(4x)} = 20$ c) $\ln(e^{5x-4}) = 11$ d) $\ln(e^{x^2-2x+1}) = 4$

Q5. Write $\frac{1}{6}[\ln x - 2(\ln y + 2 \ln z)]$ as a single logarithm.

Q6. Solve

a) $\log(x + 3)^4 = 4$ b) $10^{\log x} = 5$ c) $3^{\log_3 x} = 5$ d) $5^{\log_5 x} = 5$

Q7. If $\ln 2 = 0.7$ and $\ln 5 = 1.6$ then find $\log_2 5$.

Q8. If $\ln x = 2$ and $\ln y = 7$ then find $\ln(xy^2)$.

Q9. Solve;

a) $2^{\log_2 x + \log_2 4} = 8$ (b) $\ln(x + 1) - \ln x = \ln 2$ (c) $\log_2(x - 4) + \log_4 3 = \log_8 x$

Q10. Solve;

a) $2^{2x+1} = 8^{x-3}$ b) $10^{\log x^3} = 27$ c) $\log_x\left(\frac{9}{4}\right) = \frac{-2}{3}$ d) $\log(3x^2 + 2x - 4) = 0$

Q11. Write as a sum of logarithms;

a) $\log \sqrt[4]{a^2 b^{-3} c^{\frac{2}{3}}}$ b) $\ln\left(\frac{\sqrt{x^3}}{\sqrt[4]{y^3}}\right)$ c) $\ln\left(\frac{e^x}{e^{3x-1}}\right)$

Q12. Consider the equation $A = (1000)(1.03)^4$. Taking logarithm with base 10 of both sides, we get

$$\log A = \log 1000 + 4 \log 1.03$$

or

$$\log A = \log 10^3 + 4 \log \frac{103}{100}.$$

Therefore we obtain

$$\log A = 3 \log 10 + 4(\log 103 - 3 \log 10)$$

and so we get

$$\log A = 3 + 4(\log 103 - 2).$$

Thus we obtain

$$\log A = 4 \log 103 - 5.$$

Now making the similar computations write

a) $\log A$ from the equation; $A = (10000)(1.02)^5$.

b) $\log A$ from the equation; $A = (10^5)(1.25)^3$.