Exam

Name________________________________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

1) Let f(x) be defined by 1) _______

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$$ Where, if anywhere, is f discontinuous?

A) at $x = 0$ B) nowhere C) at $x = 6$ D) at $x = 3$ E) at $x = -3$

TRUE/FALSE. Write 'T' if the statement is true and 'F' if the statement is false.

2) True or False: If $h(x) = \dfrac{x^2 + 2x - 3}{x - 1}$ if $x \neq 1$ and $h(1) = 4$, then h is continuous at every x. 2) _______

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

3) Let f(x) have values 3) _______

$$\begin{cases} -2 & \text{if } x \leq -5 \\ \dfrac{2}{5}x & \text{if } -5 < x \leq 5 \\ 2 & \text{if } x > 5 \end{cases}$$

Where is f discontinuous?

A) at both $x = -5$ and $x = 5$
B) nowhere
C) at $x = -5$ only
D) at $x = 5$ only
E) none of the above

4) For what value of the constant c is the function $f(x) = \begin{cases} x + c & \text{if } x < 2 \\ cx^2 + 1 & \text{if } x \geq 2 \end{cases}$ 4) _______

continuous at everywhere?

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) 0

D) 1

E) none of the above

5) Let f(x) represent: 5) ________

$$f(x)=\begin{cases} x^2 + 2, & \text{for } x > 5 \\ 24, & \text{for } x = 5 \\ 4x + 7, & \text{for } x < 5 \end{cases}$$

Where is f(x) discontinuous?

A) 0
B) 2
C) 4
D) 5
E) none of the above

6) For what values of the constants c and k is the function 6) ________

$$f(x) = \begin{cases} x^3 + k & \text{if } -1 \le x \le 3 \\ c\,x & \text{if } x < -1 \text{ or } x > 3 \end{cases}$$

continuous at all x?

A) c = 7, k = -6
B) c = 6, k = 7
C) c = -7, k = 6
D) c = -6, k = -7
E) c = 7, k = -6

7) Find the equation of the tangent line to the curve $y = 2x - x^2$ at the point (2, 0). 7) ________

A) 2x - y - 4 = 0
B) 2x - y + 4 = 0
C) 2x + y + 4 = 0
D) 2x + y - 4 = 0
E) 2x + y = 0

8) If the line 4x - 9y = 0 is tangent in the first quadrant to the graph of $y = \frac{1}{3}x^3 + c$, what is the value of c? 8) ________

A) $\frac{16}{81}$ B) $\frac{81}{16}$ C) $-\frac{16}{81}$ D) $\frac{18}{81}$ E) $\frac{1}{81}$

9) Using the definition of the derivative find, the derivative of $f(x) = \sqrt{x + 2}$. 9) ________

A) $\frac{1}{\sqrt{x + 2}}$ B) $\frac{3}{2\sqrt{x + 2}}$ C) $\frac{2}{2\sqrt{x + 2}}$ D) $\frac{1}{2\sqrt{x + 2}}$ E) $\frac{1}{2\sqrt{x - 2}}$

10) Find the tangent line to the curve $y = \frac{x}{4 - x}$ at the origin. 10) ________

A) $y = -\frac{1}{4}x$ B) $y = -\frac{1}{2}x$ C) $y = \frac{1}{4}x$ D) $y = x$ E) $y = \frac{1}{2}x$

11) If $f(x) = \frac{4}{5}(\sqrt{9 - x})$, calculate f'(5) by using the definition of the derivative . 11) ________

A) $-\frac{1}{5}$ B) $-\frac{1}{10}\sqrt{5}$ C) $-\frac{4}{5}$ D) $\frac{2}{5}$ E) $\frac{1}{5}$

12) Find the slope of the line tangent to the curve $x^3 y = 1$ at the point $\left(3, \frac{1}{27}\right)$. 12) _______

A) $\frac{1}{27}$ B) $-\frac{1}{27}$ C) $\frac{1}{9}$ D) $-\frac{2}{27}$ E) $\frac{2}{27}$

13) Find the derivative of the function $f(x) = \frac{x^2 + x - 2}{x^3 + 6}$. 13) _______

A) $\frac{-x^4 + 2x^3 + 6x^2 + 12x + 6}{\left(x^3 + 6\right)^2}$

B) $\frac{-x^4 - 2x^3 - 6x^2 + 12x + 6}{\left(x^3 + 6\right)^2}$

C) $\frac{2x + 1}{3x^2}$

D) $\frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{\left(x^3 + 6\right)^2}$

E) $\frac{-x^4 - 2x^3 + 6x^2 + 12x - 6}{\left(x^3 + 6\right)^2}$

14) Calculate $f'(2)$ if $f(x) = \frac{x^2 + 3x + 2}{x^2 - 3x + 4}$. 14) _______

A) 2 B) 7 C) $\frac{1}{4}$ D) 4 E) $\frac{1}{2}$

15) Find the derivative of $f(x) = \frac{1}{\left(3x^2 + 5\right)^4}$. 15) _______

A) $-\frac{24x}{\left(3x^2 + 5\right)^5}$

B) $-\frac{4}{\left(3x^2 + 5\right)^5}$

C) $-\frac{12x}{\left(3x^2 + 5\right)^3}$

D) $\frac{24x}{\left(3x^2 + 5\right)^5}$

E) $\frac{12x}{\left(3x^2 + 5\right)^3}$

16) Differentiate the following function: $f(x) = \left(\frac{3x - 1}{x^2 + 3}\right)^2$. 16) _______

A) $\frac{2(3x - 1)(-3x^2 + 2x - 9)}{(x^2 + 3)^3}$

B) $\frac{3(3x - 1)(-3x^2 + 2x + 9)}{(x^2 + 3)^3}$

C) $\frac{2(3x - 1)(-3x^2 + 2x + 9)}{(x^2 + 3)^3}$

D) $\frac{2(3x - 1)(3x^2 + 2x + 9)}{(x^2 + 3)^3}$

E) none of the above

17) Find an equation of the line tangent to the curve $y = (x^3 + 2)^9$ at the point (-1, 1). 17) _______

A) $27y + x - 26 = 0$
B) $27x - y + 28 = 0$
C) $9x - y + 10 = 0$
D) $27x + y + 26 = 0$
E) $27y - x - 28 = 0$

18) Find $f'''(x)$ if $f(x) = \pi x^3 - 7x$. 18) _______

A) $6\pi x$ B) $6\pi x - 7$ C) 6π D) π E) 0

19) Find all local extreme values of the function $f(x) = 2x^3 + 3x^2 - 12x + 13$ and their locations. 19) _______

A) local maximum 33 at $x = -2$, local minimum 26 at $x = 1$
B) local maximum 26 at $x = -1$, local minimum 17 at $x = 2$
C) local maximum 17 at $x = -1$, local minimum 26 at $x = 2$
D) local maximum 26 at $x = -2$, local minimum 33 at $x = 1$
E) no local extrema

20) Find all local extreme values of the function $f(x) = x^3 - 6x^2 + 12x - 5$ and their locations. 20) _______

A) local maximum -61 at $x = -2$, local minimum 3 at $x = 2$
B) local maximum 3 at $x = -2$, local minimum -61 at $x = 2$
C) local maximum -61 at $x = 2$, local minimum 3 at $x = -2$
D) local maximum 3 at $x = 2$, local minimum -61 at $x = -2$
E) no local extrema

21) Determine the concavity of $f(x) = x^3 - 24x^2 + 6x + 18$ and identify any points of inflection. 21) _______

A) concave downwards on $(-\infty, 8)$, upwards on $(8, \infty)$; inflection at $x = 8$
B) concave upwards on $(-\infty, 8)$, downwards on $(8, \infty)$; inflection at $x = 8$
C) concave downwards on $(-\infty, -8)$, upwards on $(-8, \infty)$; inflection at $x = -8$
D) concave upwards on $(-\infty, -8)$, downwards on $(-8, \infty)$; inflection at $x = -8$
E) concave upwards on $(-\infty, \infty)$; no inflection points

22) Find the concavity and inflection point(s) of the function $f(x) = 7 - 6x^2 - 2x^3$. 22) ______

A) concave up on $(1, \infty)$, concave down on $(-\infty, 1)$; inflection at $x = 1$
B) concave up on $(-1, \infty)$, concave down on $(-\infty, -1)$; inflection at $x = -1$
C) concave down on $(-1, \infty)$, concave up on $(-\infty, -1)$; inflection at $x = -1$
D) concave down on $(1, \infty)$, concave up on $(-\infty, 1)$; inflection at $x = 1$
E) concave up on $(-\infty, \infty)$

23) What are the asymptotes of the graph of $y = \dfrac{2x^2 - 3}{x^2 - x - 2}$? 23) ______

A) horizontal asymptote at $y = 2$, vertical asymptotes at $x = 1$ and $x = -2$
B) horizontal asymptote at $y = 2$, vertical asymptotes at $x = -1$ and $x = 2$
C) horizontal asymptote at $y = \sqrt{\dfrac{3}{2}}$, vertical asymptotes at $x = 1$ and $x = 2$
D) oblique asymptote at $y = x - 2$, vertical asymptotes at $x = -1$ and $x = 2$
E) oblique asymptote at $y = -x - 2$, vertical asymptotes at $x = -1$ and $x = 2$

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

24) Use information obtained from f and its first two derivatives to sketch the graph of the function 24) ______________
$f(x) = x^3 - 2x^2 - 4x + 3$.

25) Find the local extrema and inflection points of the function $f(x) = \left(x^2 - 1\right)^2$ and sketch its graph. 25) ______________

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

26) Find two positive numbers whose sum is 16 such that the product of one number and the cube of the other number is a maximum. 26) ______

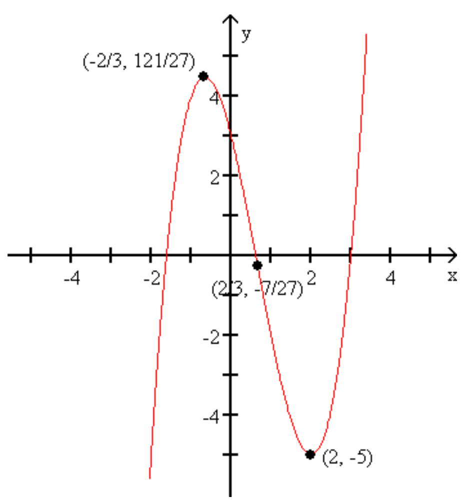
A) 3 and 13 B) 9 and 7 C) 1 and 15 D) 4 and 12 E) 8 and 8

27) Find two nonnegative numbers whose sum is 9 such that the sum of one number and the square of the other number is a maximum. 27) ______

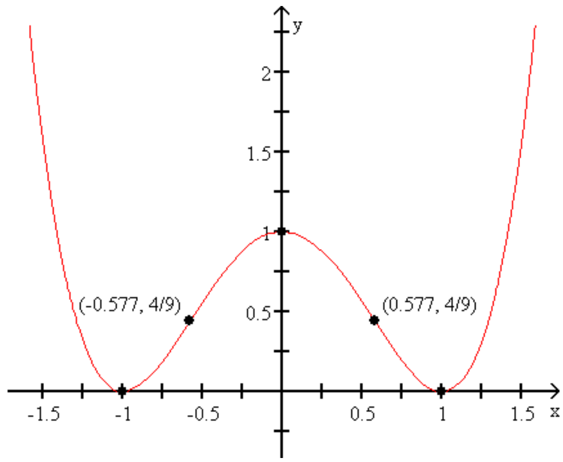
A) 1 and 8
B) 4 and 5
C) 3 and 6
D) 1/2 and 17/2
E) 0 and 9

Answer Key
Testname: UNTITLED1

1) B
2) TRUE
3) B
4) B
5) D
6) E
7) D
8) A
9) D
10) C
11) A
12) B
13) D
14) E
15) A
16) C
17) B
18) C
19) A
20) E
21) A
22) C
23) B
24) Local max (-2/3, 121/27), local min (2, -5), inflection point (2/3, -7/27).



25) Local max at (0,1), local min at (±1,0), inflections at $(\pm 1/\sqrt{3}, 4/9)$



26) D
27) E