ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Fall

FIRST MIDTERM EXAMINATION
(SAMPLE EXAM)

Question	Grade	Out of
1		
2		
3		
4		
5		
Total		

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Find the solution set of the following equations:

a) $(x-3)^2 + 5x - 11 = 0$

$x^2 - 6x + 9 + 5x - 11 = 0$
$x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$x = 2 \quad x = -1$

b) $|2x - 5| \leq 3$

$-3 \leq 2x - 5 \leq 3$
$2 \leq 2x \leq 8$
$1 \leq x \leq 4 \qquad x \in [1, 4].$

c) $\dfrac{x}{\sqrt{x}+3} - \dfrac{\sqrt{x}}{\sqrt{x}-3} = \sqrt{x} + 1$

$x(\sqrt{x}-3) - \sqrt{x}(\sqrt{x}+3) = (\sqrt{x}+1)(x-9)$
$x\sqrt{x} - 3x - x - 3\sqrt{x} = x\sqrt{x} + x - 9\sqrt{x} - 9$
$5x - 6\sqrt{x} - 9 = 0$ if $t = \sqrt{x}$
$5t^2 - 6t - 9 = 0$ then $t = \frac{6+\sqrt{216}}{10}$ and $t = \frac{6-\sqrt{216}}{10}$
But $t = \frac{6-\sqrt{216}}{10}$ negative so $x = \sqrt{\frac{6+\sqrt{216}}{10}}$

2) Let $f(x) = \dfrac{1}{\sqrt{x-1}}$ and $g(x) = \log(3x-2)$ be two functions.

a) Find the domain of f.
$x > 1$ so $Domf : (1, \infty)$

b) Find the domain of g.
$3x - 2 > 0 \implies x > \dfrac{2}{3}$ so $Domg : (\frac{2}{3}, \infty)$

c) Find $f \circ g(x)$.
$f \circ g(x) = f(g(x)) = \dfrac{1}{\sqrt{g(x) - 1}} = \dfrac{1}{\sqrt{\log(3x-2) - 1}}$

d) Find the domain of $f \circ g(x)$. (BONUS)
$\log(3x-2) - 1 > 0 \quad \log(3x-2) > 1 \quad 3x - 2 > 10 \quad x > 4$

3) Find the equation of the line

a) passing through the points $(-3, 5)$ and $(1, -3)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-3-5}{1-(-3)} = -2$
Equation of the line $y - y_1 = m(x - x_1) \implies y - 5 = -2(x - (-3)) \quad y = -2x - 1$

b) passing through the points $(3,-2)$ and parallel to the line $3x-2y=5$.

Since $y=\frac{3}{2}x-\frac{5}{2}$, slope of the line $3x-2y=5$ is $\frac{3}{2}$. So the slope of the line is also $\frac{3}{2}$.

Equation of the line $y-y_1=m(x-x_1) \implies y-(-2)=\frac{3}{2}(x-3) \quad 2y-3x-13=0$

c) passing through the points $(-2,0)$ and perpendicular to the line $x+5y=4$.

Since $y=\frac{-1}{5}x+\frac{4}{5}$, slope of the line $3x-2y=5$ is $\frac{-1}{5}$.

Since the lines are perpendicular $m.\frac{-1}{5}=-1 \quad m=5$ so the slope of the line is 5.

Equation of the line $y-y_1=m(x-x_1) \implies y-0=5(x-(-2)) \quad y=5x+10$.

4) Consider the function $y=-x^2+2x-5$.

a) Find the vertex, x-intercept(s) and y-intercept(s). (If any)

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) \implies \frac{-b}{a}=\frac{-2}{-2}=1$ and $f(1)=-4$. So vertex:(1,-4)

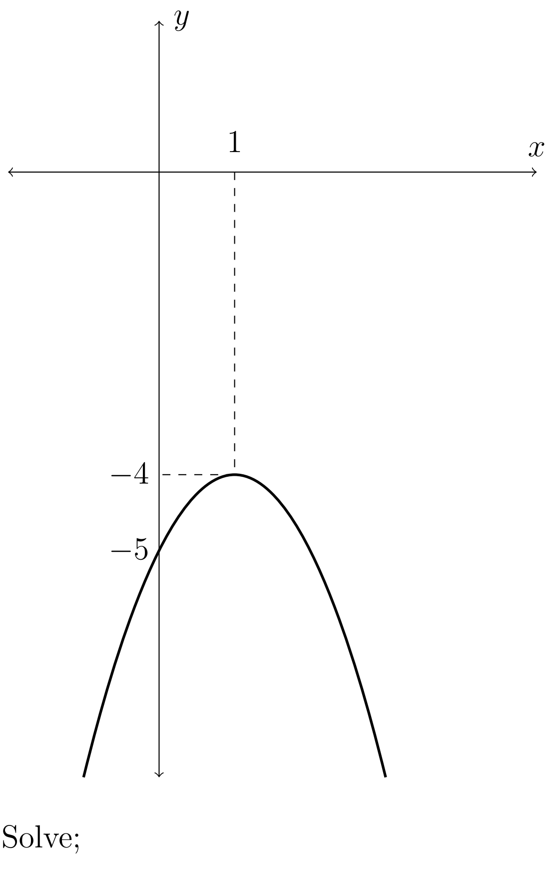
y-intercept: $(0,-5)$

x-intercept: Since $b^2-4ac=4-20=-16<0$ no x-intercept.

b) Find *Dom* f and *Range* f.

Dom $f:\mathbb{R}$ and *Range* $f:(-\infty,-4]$

c) Sketch the graph.



5) Solve;

a) $\log_x(3-x)+\log_x(2+x)=2$.

$\log_x((3-x)(2+x))=2$

$(3-x)(2+x)=x^2$

$2x^2 - x - 6 = 0 \implies (x-2)(2x+3) = 0 \implies x = 2$ and $x = \dfrac{-3}{2}$.
But x must be positive so $x = 2$

b) $\log(x+5) + \log(2x-3) = \log(x+2) + 1$.

$\log\left(\dfrac{(x+5)(2x-3)}{x+2}\right) = 1 \implies \dfrac{(x+5)(2x-3)}{x+2} = 10$
$2x^2 + 10x - 3x - 15 = 10x + 20 \implies 2x^2 - 3x - 35 = 0 \implies (2x+7)(x-5) = 0$
Then $x = 5$ and $x = \dfrac{-7}{2}$ but $x = \dfrac{-7}{2}$ is not in the domain of $\log(2x-3)$ so the only solution is $x = 5$.