ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Fall

FIRST MIDTERM EXAMINATION
(SAMPLE EXAM)

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

Question	Grade	Out of
1		
2		
3		
4		
Total		

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 4 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) Solve the following equalities and inequalities for the unknown variable x. Express the solutions clearly.

a) (7 points) $\dfrac{1}{\sqrt{x+1}-\sqrt{x}} - 1 = 1$

$$\frac{\sqrt{x+1}+\sqrt{x}}{(\sqrt{x+1}-\sqrt{x})(\sqrt{x+1}+\sqrt{x})} = 2$$

$$\frac{\sqrt{x+1}+\sqrt{x}}{x+1-x} = 2$$

$$\sqrt{x+1}+\sqrt{x} = 2$$

$$(\sqrt{x+1})^2 = (2-\sqrt{x})^2$$

$$x+1 = 4-4\sqrt{x}+x \Rightarrow \sqrt{x} = \frac{3}{4}$$

$$\boxed{x = \frac{9}{16}} \qquad S.S: \left\{\frac{9}{16}\right\} //$$

b) (7 points) $1+|x-2| \leq 5$

$$|x+2| \leq 4$$

$$x-2 \leq 4 \quad \text{OR} \quad -(x-2) \leq 4$$

$$\boxed{x \leq 6} \qquad x-2 \geq -4$$

$$\boxed{x \geq -2}$$

So, $\boxed{-2 \leq x \leq 6}$ // means $x \in [-2,6]$ //

c) (8 points) $e^{\ln(x+3)-2\ln(x)} = 2$

$$e^{\ln\left(\frac{x+3}{x^2}\right)} = 2$$

$$\frac{x+3}{x^2} = 2$$

$$2x^2 - x - 3 = 0$$

$$(2x-3)(x+1) = 0 \Rightarrow \boxed{x = \frac{3}{2}}, \boxed{x = -1}$$

$x=-1$ makes the logarithm undefined!

$$S.S: \left\{\frac{3}{2}\right\} //$$

d) (8 points) $3 = 2 + 10e^{-2x}$

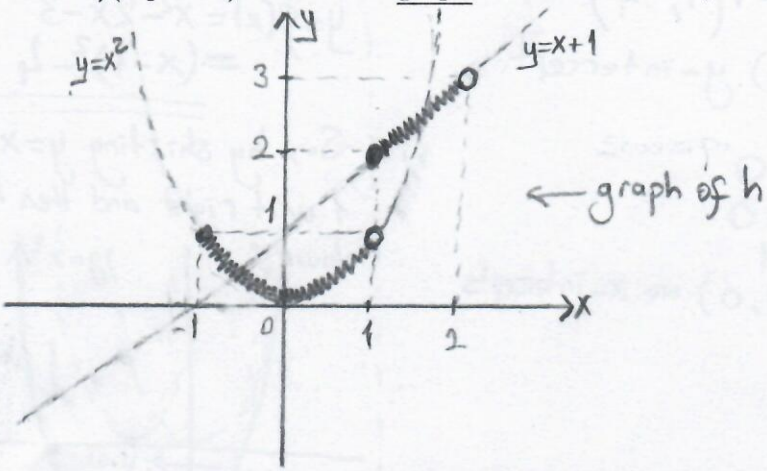
$$\frac{1}{10} = e^{-2x}$$

$$\ln\left(\frac{1}{10}\right) = \ln e^{-2x}$$

$$0 - \ln 10 = (-2x)\cdot 1 \Rightarrow \boxed{x = \frac{(\ln 10)}{2}} \approx 1.1513 //$$

2) For the given functions $f(x) = \dfrac{1}{x^2-1}$, $g(x) = 5$ and $h(x) = \begin{cases} x^2, & \text{if } -1 \le x < 1 \\ x+1, & \text{if } 1 \le x < 2 \end{cases}$,

a)(6 points) Sketch the graph of the function h,



b)(4 points) Find the Domain and Range of the function h,

$D(h): [-1,1) \cup [1,2)$ means $[-1,2)$

$R(h): [0,1) \cup [2,3)$

c)(4 points) Calculate (if possible) $(h-g)(1)$ and $(\frac{f}{g})(1)$.

$(h-g)(1) = h(1) - g(1) = (1+1) - 5 = \boxed{-3}$ //

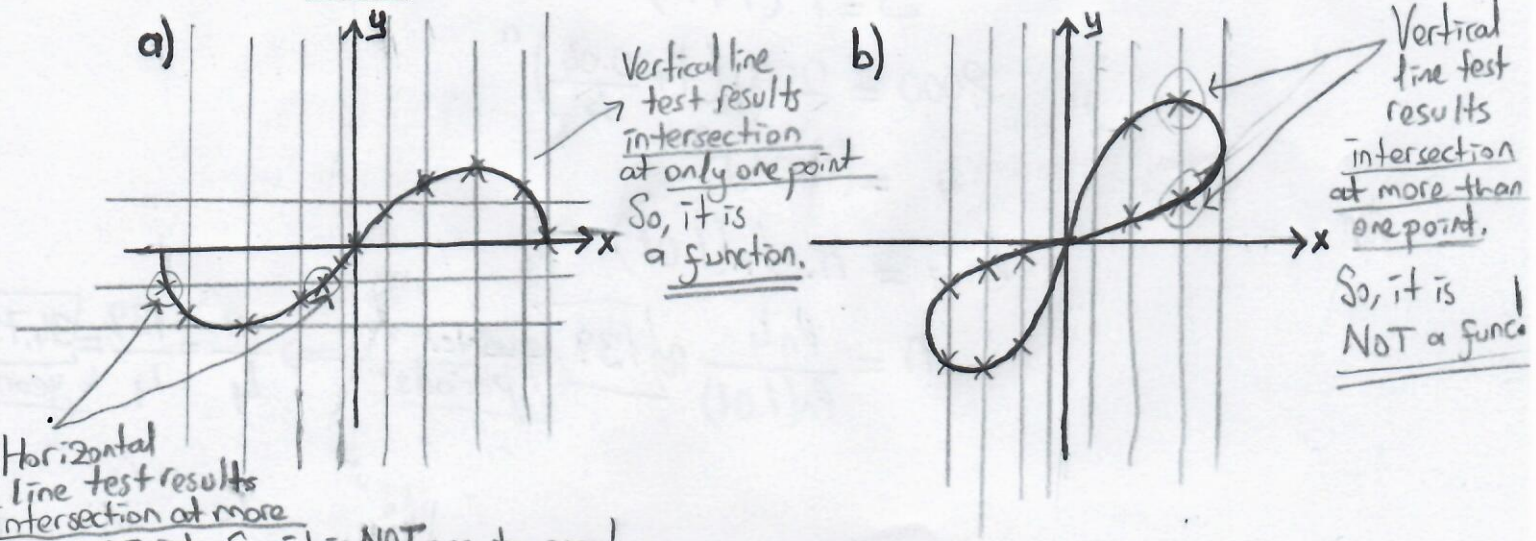
$(\frac{f}{g})(-1) =$

d)(6 points) Calculate (if possible) $(h \circ f)(0)$ and $(g \circ h)(0)$.

$(h \circ f)(0) = h(f(0)) = h(-1) = (-1)^2 = \boxed{1}$ //

$(g \circ h)(0) = g(h(0)) = g(0) = \boxed{5}$ //

3) (10 points) Do the following graphs define a function of x ? If so, are they one-to-one functions? Explain your answer clearly (*Yes or No answer is not enough*).



4) **a)**(15 points) Sketch clearly the graph of quadratic function $y = f(x) = x^2 - 2x - 3$. Explain your steps.

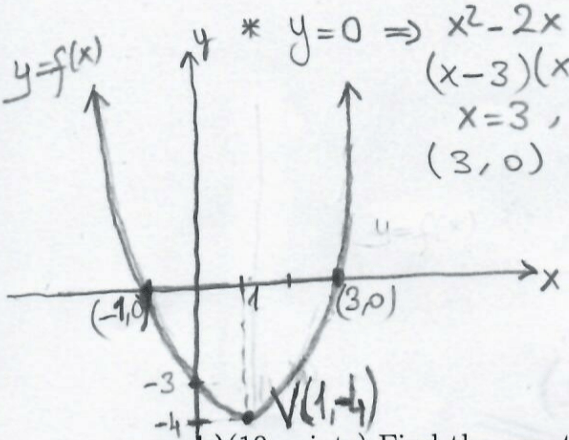
$a=1, b=-2, c=-3$

1st way:

Vertex $V = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = V(1,-4)$

Intercepts: * $y = c = -3$ $(0,-3)$ y-intercept
$(x=0 \Rightarrow y=-3)$

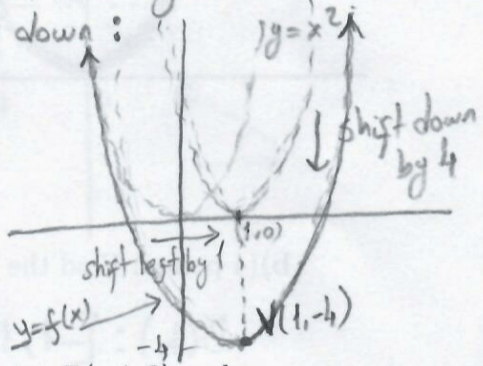
* $y=0 \Rightarrow x^2-2x-3=0$
$(x-3)(x+1)=0$
$x=3$, $x=-1$
$(3,0)$ & $(-1,0)$ are x-intercepts



2nd way:

$y = f(x) = x^2 - 2x - 3$
$= (x-1)^2 - 4$

So, by shifting $y=x^2$ 1 unit right and then 4 units down:



b)(10 points) Find the equation of the line passing through the point $P(-1,3)$ and perpendicular to the line $3y - 5x + 1 = 0$.

Eqn. of the line: $y - y_1 = m_1(x - x_1)$ where $(x_1, y_1) = (-1,3)$ and m_1 is unknown $\Rightarrow$

Perpendicular lines have $m_1 \cdot m_2 = -1$

So $3y - 5x + 1 = 0$

$y = \frac{5}{3}x + \frac{1}{3}$

m_2

$m_1 \cdot m_2 = -1 \Rightarrow m_1 = \frac{-3}{5}$

So $y - 3 = \left(\frac{-3}{5}\right)(x+1)$

$y = \frac{-3}{5}x + \frac{12}{5}$ //

eqn. of the given line.