ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Fall

FIRST MIDTERM EXAMINATION
(SAMPLE EXAM)

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

Question	Grade	Out of
1		
2		
3		
4		
5		
6		
Total		

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1) For the case-defined function $f(x) = \begin{cases} x+5, & \text{if } -10 \le x < 5 \\ 3, & \text{if } 5 \le x \le 20 \end{cases}$

a)(8 points) Find the domain and range of the function $f(x)$.

for $-10 \le x < 5 \Rightarrow -5 \le x+5 < 10$

$-5 \le f(x) < 10$

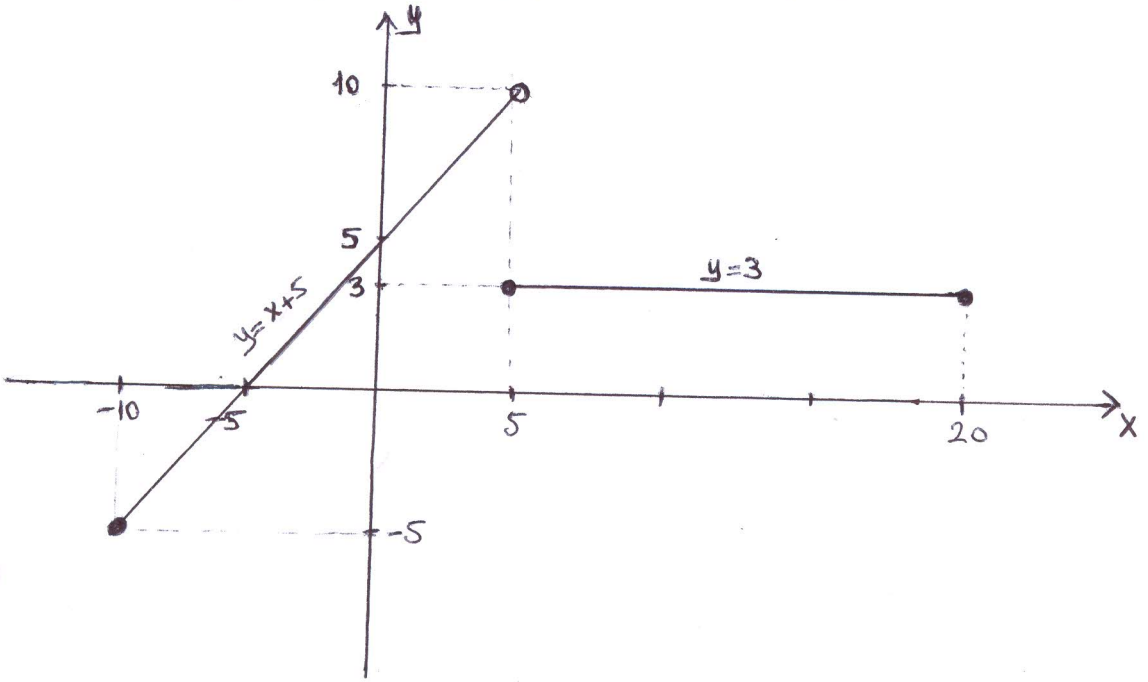
for $5 \le x \le 20 \Rightarrow f(x) = 3$ constantly

Therefore,

Domain(f): $[-10, 20]$

Range(f): $[-5, 10)$

b)(12 points) Sketch the graph of the function $f(x)$.



Graph of $y=f(x)$

2) For the function $f(x) = x^2 + x - 12$

a)(5 points) Find the domain and the range of the function.

This is a polynomial function, so it is defined for all $x \in \mathbb{R}$

Thus Domain $(f) = \mathbb{R}$ (OR $(-\infty, \infty)$)

$f(x) = x^2 + x - 12 = \left(x + \frac{1}{2}\right)^2 - \frac{49}{4}$ so for $x = -\frac{1}{2}$ $f(x)$ has a minimum value, $f\left(-\frac{1}{2}\right) = -\frac{49}{4}$. Range $(f) = \left[-\frac{49}{4}, \infty\right)$

b)(8 points) Find the intercepts and the vertex for the function.

Intercepts: $x = 0 \Rightarrow y = -12$ y-intercept is the point $(0, -12)$

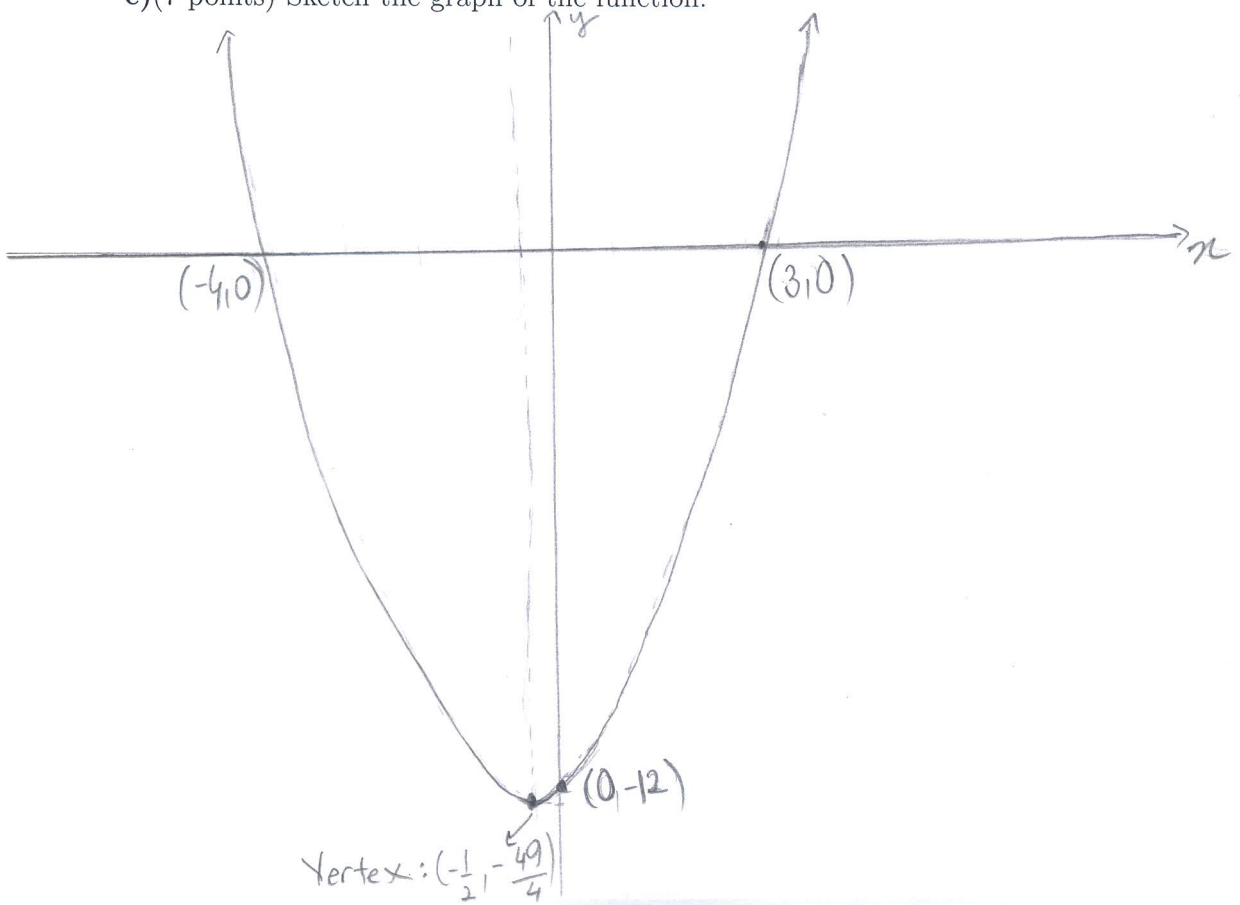
$y = 0 \Rightarrow x^2 + x - 12 = (x+4)(x-3) = 0 \Rightarrow x = -4, x = 3$

x-intercepts are the points $(-4, 0)$ and $(3, 0)$

Vertex: For $f(x)$, $a = 1$, $b = 1$, $c = -12$

Vertex is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right) = \left(-\frac{1}{2}, -\frac{49}{4}\right)$

c)(7 points) Sketch the graph of the function.



3) a)(7 points) Find the equation for the line passing through the points $P(-2,1)$ and $Q(-1,3)$.

$P(-2,1) = (x_1, y_1)$ $\quad Q(-1,3) = (x_2, y_2)$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\frac{y_2 - y_1}{x_2 - x_1}$ → slope m

$$y - 1 = \left(\frac{3-1}{-1+2}\right)(x+2)$$

$$\boxed{y = 2x + 5} \quad (\text{or } y - 2x - 5 = 0)$$

b)(7 points) Find the equation for the line passing through the point $P(-2,2)$ and perpendicular to the line $3x - y = 4$.

The line $3x - y = 4$ has the slope $m = 3$ (since $3x - y = 4$, $y = 3x - 4$, $y = mx + b$).

Our line is perpendicular to that one, so its slope is $\frac{-1}{m}$ and its equation becomes

$$y - y_1 = \left(\frac{-1}{m}\right)(x - x_1), \text{ since } P(-2,2) = (x_1, y_1)$$

$$y - 2 = \left(\frac{-1}{3}\right)(x+2)$$

$$\boxed{y = -\frac{1}{3}x + \frac{4}{3}} \quad (\text{or } 3y + x - 4 = 0)$$

c)(6 points) Sketch the graph of the line given by the equation $\frac{x}{2} - \frac{y}{3} = 5$.

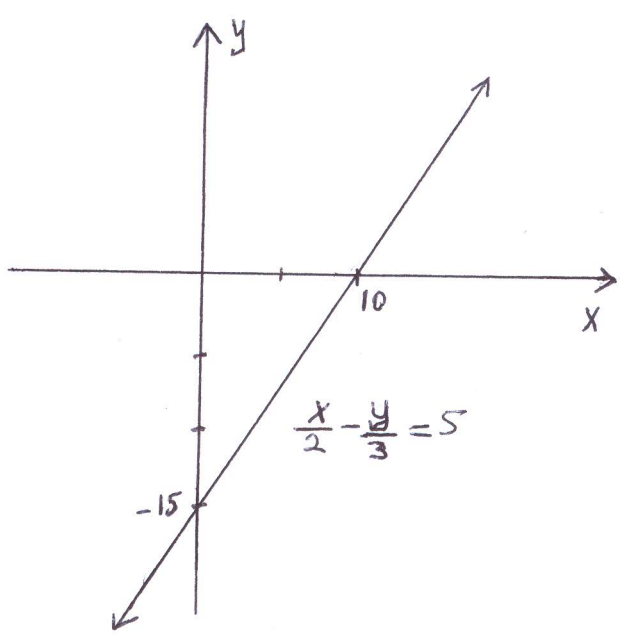
$$\frac{x}{2} - \frac{y}{3} = 5 \Rightarrow 3x - 2y = 30 \Rightarrow y = \frac{3}{2}x - 15 \quad (\text{in the form: } y = mx + b)$$

$m = \text{slope} = \frac{3}{2}$, positive slope!

Intercepts:

$x = 0 \Rightarrow y = -15$ so $(0, -15)$ y-intercept

$y = 0 \Rightarrow x = 10$ so $(10, 0)$ x-intercept



4) a)(5 points) Simplify the expression $10^{3\log x+6\log y}$.

$3\log x + 6\log y = \log x^3 + \log y^6 = \log(x^3y^6)$

$10^{3\log x + 6\log y} = 10^{\log(x^3y^6)} = x^3y^6$

b)(5+5=10 points) Solve the following equations for x.

i) $\ln\sqrt[3]{x+8} = 2$

$\ln\sqrt[3]{x+8} = 2 \Rightarrow \sqrt[3]{x+8} = e^2 \Rightarrow x+8 = e^6 \Rightarrow x = e^6 - 8$

ii) $e^{\ln(x-2)+\ln(x+4)} = 5$ $\quad \ln(x-2) + \ln(x+4) = \ln[(x-2)(x+4)]$

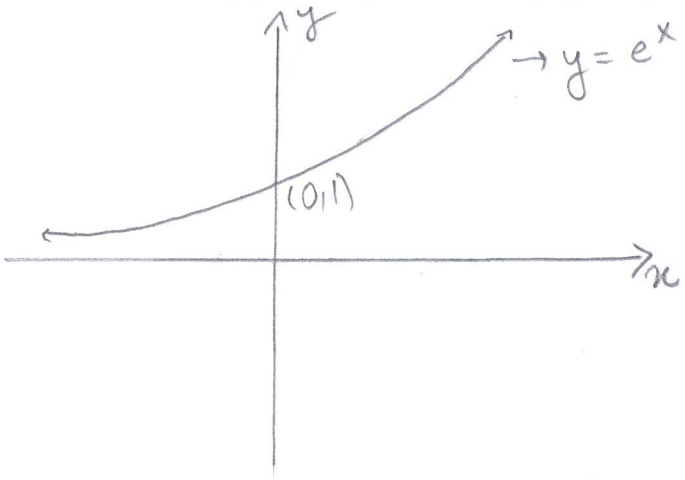
$e^{\ln(x-2)+\ln(x+4)} = e^{\ln[(x-2)(x+4)]} = (x-2)(x+4) = 5$

$\Rightarrow x^2 + 2x - 13 = 0 \Rightarrow x_{1,2} = \frac{-b \mp \sqrt{b^2-4ac}}{2a} \Rightarrow x_1 = -1+\sqrt{14}$, $x_2 = -1-\sqrt{14}$

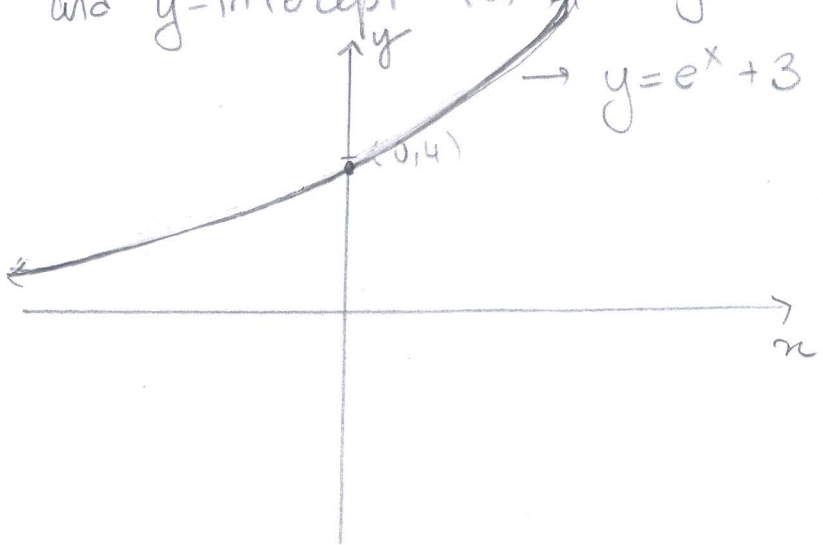
For $x_2 = -1-\sqrt{14}$ $\ln(x-2)$ and $\ln(x+4)$ are not defined

so $x_1 = -1+\sqrt{14}$ is the only solution.

c) (5 points) Sketch the graph of $y = f(x) = e^x + 3$.



Using the graph on the left and y-intercept (0,4) we get

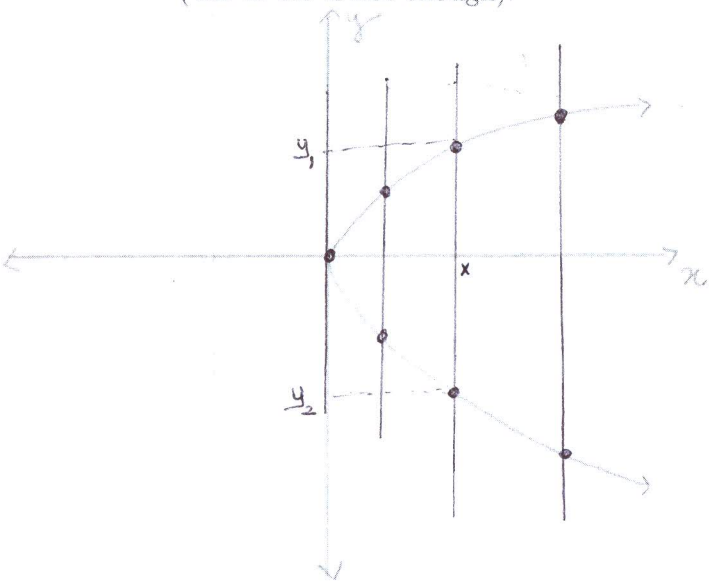


For $y = f(x) = e^x + 3$

$x = 0 \quad y = e^0 + 3 = 4$

There is no x-intercept

5) a)(8 points) Does the graph in the following define a function of x? Explain your answer (Yes or No is not enough).



Applying the vertical line test results intersection at most two points, so two different images y_1 and y_2 for the same value of x.

$\therefore$ It is not a function of x.

b)(12 points) For the functions $f(x) = x^3 - 1$ and $g(x) = \begin{cases} \sqrt{x}, & \text{if } 0 \le x \le 1 \\ 2 - x, & \text{if } 1 < x \le 3 \end{cases}$

Calculate $(f \circ g)(0)$, $(g \circ f)(\frac{3}{2})$, $(f \circ f^{-1})(2)$.

$(f \circ g)(0) = f(g(0)) = f(\sqrt{0}) = 0^3 - 1 = \boxed{-1}$

$(g \circ f)(\frac{3}{2}) = g(f(\frac{3}{2})) = g((\frac{3}{2})^3 - 1) = g(\frac{19}{8}) = 2 - \frac{19}{8} = \boxed{\frac{-3}{8}}$

$(f \circ f^{-1})(2) = I(2) = \boxed{2}$ (since $(f \circ f^{-1})(x) = \underbrace{I(x) = x}_{\text{identity function}}$)

6) (Bonus)

a)(10 points) Solve the equation $5xe^{-x} + x^2e^{-x} = 0$ for x.

$5xe^{-x} + x^2e^{-x} = e^{-x}(5x + x^2) = 0$

as $e^{-x} > 0$ for all $x \in \mathbb{R}$ we have $x^2 + 5x = 0$

$\Rightarrow \quad x(x+5) = 0 \quad \Rightarrow \quad x = 0, \; x = -5$

Solution set is $\{-5, 0\}$

b)(5 points) Given the function $f(x) = 1 + |x-2|$. Write this function as a case-defined function.

$$|x-2| = \begin{cases} x-2 & \text{if } x \geq 2 \\ 2-x & \text{if } x < 2 \end{cases}$$

So $f(x) = 1 + x - 2 = x - 1$ if $x \geq 2$ and

$f(x) = 1 + 2 - x = 3 - x$ if $x < 2$.

As a case-defined function

$$f(x) = \begin{cases} x-1 & \text{if } x \geq 2 \\ 3-x & \text{if } x < 2 \end{cases}$$

Extra: Let A be the grade that you will get from this exam. Make a guess for your grade. If your guess is in the open interval $(A-5, A+5)$ you will get extra 5 points. Write your guess in the box. What is your guess?

$\square$