ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 105 - Business Mathematics I

2018-2019 Fall

FIRST MIDTERM EXAMINATION
(SAMPLE EXAM)

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 90 minutes

Question	Grade	Out of
1		
2		
3		
4		
5		
Total		

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

1. Find the solution sets of the following expressions. Express the solutions clearly.

a) (5 points) $(x+1)^2 - 3x - 2 = 0$

$(x+1)^2 - 3x - 2 = x^2 + 2x + 1 - 3x - 2 = x^2 - x - 1$

Hence we have $x^2 - x - 1 = 0$. Use quadratic formula:

$a = 1,\ b = -1,\ c = -1 \Rightarrow \Delta = b^2 - 4ac = 1 - 4(1)(-1) = 5$

$x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{1+\sqrt{5}}{2}$, $x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{1-\sqrt{5}}{2} \Rightarrow$ Soln set $= \{x_1, x_2\}$

b) (5 points) $\sqrt{2x+6} = x - 1$

$(\sqrt{2x+6})^2 = (x-1)^2 \Rightarrow 2x + 6 = x^2 - 2x + 1$

$\Rightarrow x^2 - 4x - 5 = 0 \Rightarrow (x-5)(x+1) = 0 \Rightarrow x_1 = -1$ or $x_2 = 5$

$$\left.\begin{array}{l}\text{Substitute } x=-1 \Rightarrow \sqrt{-2+6} \neq -2 \\ \text{Substitute } x=5 \Rightarrow \sqrt{10+6} = 4 = 5-1\end{array}\right\} \text{Soln set} = \{5\}$$

c) (5 points) $|4x - 3| + 2 < 11$

$|4x-3| + 2 < 11 \Rightarrow |4x-3| < 9 \Rightarrow -9 < 4x - 3 < 9$

$\Rightarrow -6 < 4x < 12 \Rightarrow -\frac{6}{4} < x < 3 \Rightarrow -\frac{3}{2} < x < 3$

Soln set $= \left(-\frac{3}{2}, 3\right)$

d) (5 points) $\left|\frac{3x-7}{2}\right| \geq 4$

$\left|\frac{3x-7}{2}\right| \geq 4 \Rightarrow \frac{|3x-7|}{2} \geq 4 \Rightarrow |3x-7| \geq 8$

Hence, $3x - 7 \geq 8$ or $3x - 7 \leq -8$

$\Downarrow$ $\quad\quad$ $\Downarrow$

$3x \geq 15$ $\quad\quad$ $3x \leq -1$

$\Downarrow$ $\quad\quad$ $\Downarrow$

$x \geq 5$ or $x \leq -\frac{1}{3}$

Soln set $= \left(-\infty, -\frac{1}{3}\right] \cup [5, \infty)$

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2. Consider the functions $f(x) = \sqrt{9-x^2}$ and $g(x) = \dfrac{x}{x^2-4}$.

a) (8 points) Find the domains of the functions $f(x)$ and $g(x)$.

$f(x) = \sqrt{9-x^2} \Rightarrow 9-x^2 \geq 0$

$\Rightarrow (3-x)(3+x) \geq 0$

	$-\infty$	-3	3	∞
$3+x$	$-$	ϕ $+$	$+$	
$3-x$	$+$	$+$	ϕ $-$	
$9-x^2$	$-$	ϕ $+$	ϕ $-$	

Domain of f is $[-3,3]$.

$g(x) = \dfrac{x}{x^2-4}$; $x^2-4=0 \Rightarrow (x-2)(x+2)=0$

$\Rightarrow x = \mp 2$

Hence, when $x = \mp 2$, $g(x)$ is undefined

Domain of g is $= \mathbb{R} \setminus \{-2, 2\}$

b) (4 points) Evaluate $(f-g)(1)$ and $(fg)(-1)$.

$(f-g)(1) = f(1) - g(1) = \sqrt{9-1^2} - \dfrac{1}{1^2-4} = \sqrt{8} - \left(-\dfrac{1}{3}\right) = \sqrt{8} + \dfrac{1}{3}$

$(f\cdot g)(-1) = f(-1)\cdot g(-1) = \left(\sqrt{9-(-1)^2}\right)\left(\dfrac{-1}{(-1)^2-4}\right) = \sqrt{8}\left(+\dfrac{1}{3}\right) = +\dfrac{\sqrt{8}}{3}$

c) (4 points) Evaluate $(f \circ g)(-1)$ and $(g \circ f)(2)$.

$(f\circ g)(-1) = f(g(-1)) = f\left(\dfrac{-1}{(-1)^2-4}\right) = f\left(\dfrac{1}{3}\right) = \sqrt{9-\left(\dfrac{1}{3}\right)^2} = \sqrt{9-\dfrac{1}{9}} = \dfrac{\sqrt{80}}{3}$

$(g\circ f)(2) = g(f(2)) = g\left(\sqrt{9-2^2}\right) = g(\sqrt{5}) = \dfrac{\sqrt{5}}{(\sqrt{5})^2-4} = \dfrac{\sqrt{5}}{5-4} = \sqrt{5}$

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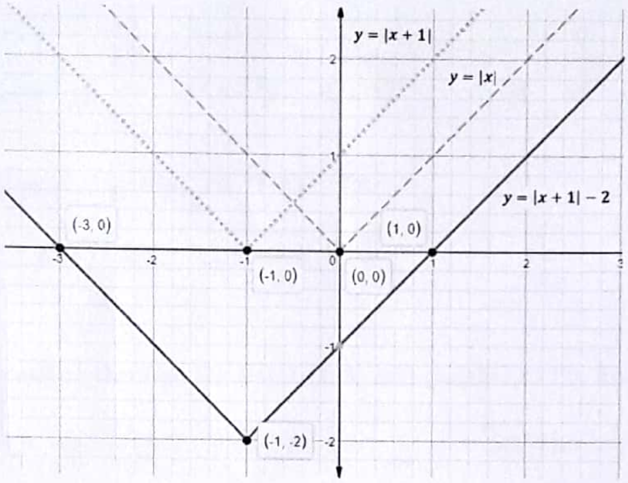
3. a) (6 points) Use the graph of the function $f(x) = |x|$ and transformation techniques to sketch the graph of the function $g(x) = |x+1| - 2$.



b) Consider the function $f(x) = -x^2 + 4x + 12$.

b1) (5 points) Find the vertex, x-intercept(s) and y-intercept(s) of $f(x)$ (If any).

Vertex: $a=-1,\ b=4,\ c=12 \Rightarrow -\frac{b}{2a} = -\frac{4}{-2} = 2 \Rightarrow f(2) = -2^2+8+12$
$= 16$

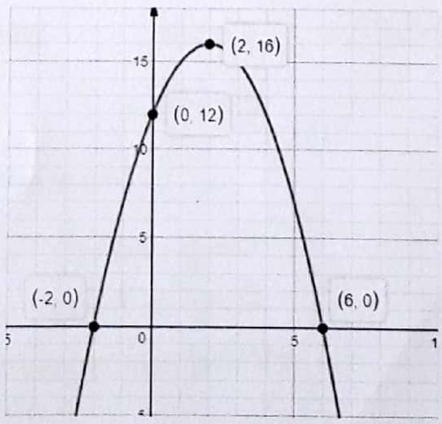
$\Rightarrow (2,16)$ is vertex

y-int: $x=0 \Rightarrow y=f(0) = 12 \Rightarrow (0,12)$ is y-int.

x-int: $f(x)=0 \Rightarrow -x^2+4x+12=0 \Rightarrow (-x+6)(x+2)=0 \Rightarrow x_1=-2,\ x_2=6$

$\Rightarrow (-2,0)$ and $(6,0)$ are x-intercepts.

b2) (3 points) Sketch the graph of f.



b3) (2 points) Find the domain and range of $f(x)$.

Domain $= \mathbb{R}$

Range $= (-\infty, 16]$

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4. a) Find the equation of the line

a1) (5 points) passing through the points $(-3, -2)$ and $(1, 4)$.

$$* \quad \left.\begin{array}{l}(x_1, y_1) = (-3, -2) \\ (x_2, y_2) = (1, 4)\end{array}\right\} \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-4}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$$

$*$ $(x_0, y_0) = (1, 4)$ and $(y - y_0) = m(x - x_0)$

$\Rightarrow (y-4) = \frac{3}{2}(x-1) \Rightarrow y = \frac{3}{2}x + \frac{5}{2}$

a2) (5 points) passing through the point $(1, 3)$ and parallel to the line $2y + 6x - 8 = 0$.

$*$ $2y + 6x - 8 = 0 \Rightarrow 2y = -6x + 8 \Rightarrow y = -3x + 4 \Rightarrow$ slope is -3

Since line is // to $2y + 6x - 8 = 0$, $m = -3$.

$*$ $(x_0, y_0) = (1, 3)$, $m = -3$ and $(y - y_0) = m(x - x_0)$

$\Rightarrow (y-3) = -3(x-1) \Rightarrow y = -3x + 6$.

b) Consider the piecewise-defined function $f(x) = \begin{cases} 2x, & \text{if } 0 \le x < 2 \\ x+2, & \text{if } 2 \le x < 6 \end{cases}$

b1) (5 points) Find the values of $f(-1)$, $f(1)$, $f(2)$, $f(5)$ and $f(6)$.

Domain of f is $[0, 6)$

$f(-1)$ is undefined $\quad f(2) = 4 \quad f(6)$ is undefined

$f(1) = 2 \quad f(5) = 7$

b2) (5 points) Sketch the graph of f.

line 1: $y = 2x \Rightarrow x = 0 \Rightarrow y = 0$

$x = 1 \Rightarrow y = 2$

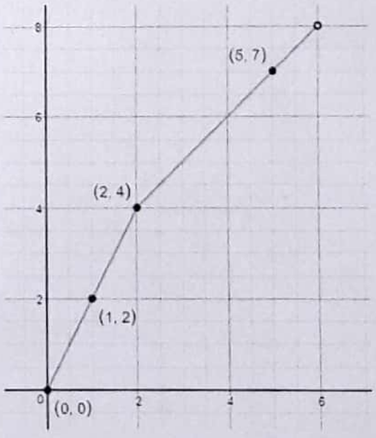
line 1 passes through the points $(0,0)$ and $(1,2)$

line 2: $y = x + 2 \Rightarrow x = 2 \Rightarrow y = 4$

$x = 5 \Rightarrow y = 7$

line 2 passes through the points $(2,4)$ and $(5,7)$

$\Rightarrow$



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5. a) Simplify the following expressions.

a1) (4 points) $\log_2\left(\dfrac{x^2(x+2)^2}{(x+1)^5}\right)$

$$\log_2\left(\frac{x^2(x+2)^2}{(x+1)^5}\right) = \log_2\left(x^2(x+2)^2\right) - \log_2\left((x+1)^5\right)$$

$$= \log_2(x^2) + \log_2((x+2)^2) - \log_2((x+1)^5)$$

$$= 2\log_2(x) + 2\log_2(x+2) - 5\log_2(x+1)$$

a2) (4 points) $\ln\left(\dfrac{\sqrt{4e^3}}{2}\right) + \log_{16}(4) - 3^{\log_3(5)} + \log_5(125)$

$$\ln\left(\frac{\sqrt{4e^3}}{2}\right) + \log_{16}(4) - 3^{\log_3(5)} + \log_5(125)$$

$$= \ln(\sqrt{4}\cdot e^{3/2}) - \ln(2) + \frac{\log_2(4)}{\log_2(16)} - 5 + \log_5(5^3)$$

$$= \cancel{\ln(2)} + \ln(e^{3/2}) - \cancel{\ln(2)} + \frac{2}{4} - 5 + 3$$

$$= \frac{3}{2} + \frac{1}{2} - 2 = 0.$$

b) Solve the following equalities.

b1) (4 points) $\log(98 - x + x^2) = 2$

$$10^2 = 98 - x + x^2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x-2)(x+1) = 0$$

$$\Rightarrow x_1 = -1 \text{ or } x_2 = 2$$

b2) (4 points) $2e^{3x} - 5 = 3$

$$2e^{3x} - 5 = 3 \Rightarrow 2e^{3x} = 8 \Rightarrow e^{3x} = 4$$

$$\Rightarrow \ln(e^{3x}) = \ln(4) \Rightarrow 3x = \ln(4) \Rightarrow x = \frac{\ln(4)}{3}$$

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